Mortgage Choice and the Credit Guarantee

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Abstract

I analyze the general equilibrium effects of government-sponsored enterprises' credit guarantees for fixed-rate mortgages. I develop a macroeconomic model where borrowers choose between fixed-rate mortgages (FRMs) and adjustable-rate mortgages (ARMs) provided by a constrained financial intermediary. Relative to FRMs, ARMs typically have lower required payments during recessions, thus generating lower and less cyclical default rates. Since the intermediary prices the exposure to credit risk, borrowers choose 60% of ARMs in an economy without credit guarantees. This outcome aligns with some European economies where ARMs are prevalent and intermediaries maintain unhedged balance sheets against credit risk. With existing guarantees, the intermediary does not bear FRMs' credit risk, resulting in a high insensitivity of the FRM rate to borrower leverage. As observed in the US, this leads to a 70% FRM share. Compared to the economy without guarantees, mortgage default rates are higher, while intermediary equity, borrower consumption and house price volatility increase. The difference in the general equilibrium impact of guarantees crucially relies on my model's novel feature of endogenizing the mortgage choice.

Keywords: Housing market, mortgage contracts, equilibrium model.

JEL: E2, E4, E6, G2, G5

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1 Introduction and Motivation

A unique feature of the mortgage market in the US is the substantial prevalence of prepayable 30-year fixed-rate mortgages (FRMs). Currently, approximately 90% of newly originated mortgages feature a fixed rate for the entire duration of the loan[1]. Even amidst a significant increase in mortgage rates in 2022, the share of adjustable-rate mortgages (ARMs) in weekly mortgage applications has consistently remained below 10%[2]. A second notable characteristic of the US mortgage market is the large share of mortgages funded through Agency Mortgage-Backed Securities (MBS). These MBSs are issued by Fannie Mae and Freddie Mac, collectively known as the government-sponsored enterprises (GSEs), as well as Ginnie Mae[3]. These agencies play a crucial role by providing a public credit risk guarantee on the MBS which is backed by the US government, offering investors protection against credit risk[4]. Since 2008, the annual fraction of mortgage origination volume funded with some form of public credit risk guarantee has reached as high as 90% and has never fallen below 60%[5,6]. In this paper I analyze the general equilibrium effects of eliminating the credit guarantee on FRMs when the mortgage choice is endogenous.

In the aftermath of the 2008 financial crisis, several housing finance reforms have been proposed to either partially or completely eliminate the credit risk guarantee[7]. These reform proposals raise questions about their potential repercussions on house prices, the stable provision of mortgage credit, financial stability, and, ultimately, overall welfare. I address these questions by considering the endogenous choice between FRMs and ARMs, providing a novel contribution to the existing literature.

I introduce a quantitative general equilibrium model of the mortgage and housing markets in which the pricing of credit offered by the financial sector directly affects household portfolio choices. In the model,

[1] More than 75% comprise 30-year fixed-rate mortgages, while the remainder primarily consists of 15-year fixed-rate mortgages. Refer to section 7.1 for a historical context on Adjustable-Rate Mortgages (ARMs) in the US, providing some definitions that illustrate the declining significance of ARMs over the past several decades in the US.

[2] See (Goodman et al., 2023).

[3] For a detailed exploration of these agencies, please refer to section 7.2.

[4] The GSEs (Fannie Mae and Freddie Mac) typically account for about two-thirds of mortgage originations with some form of public credit risk guarantee. These institutions have been in conservatorship since 2008. The Government National Mortgage Association, or Ginnie Mae, comprises the remaining third and holds an explicit guarantee from the US government. For detailed institutional distinctions between Ginnie Mae and Fannie Mae/Freddie Mac, see section 7.2.

[5] For a detailed examination of the size of the US mortgage market and the impact of the GSEs and Ginnie Mae, refer to Section 7.2.3.

[6] In section 7.4, I provide evidence concerning the mortgage market in the UK, which stands as the largest mortgage market in Europe. Notably, this market exhibits similarities to the US in terms of homeownership rates and the proportion of homeowners holding mortgages. I show that in the UK, mortgagors mainly originate short-term mortgages (relative to the US), while financial intermediaries' balance sheets serve as the primary funding source. Building on this comparison, Section 7.5 extends the analysis to encompass a broader set of European economies. The findings in this section reinforce the earlier observations, highlighting the uniqueness of the US mortgage market within the global context.

[7] For a more detailed exploration of these proposed reforms, refer to section 7.2.4.

constrained financial intermediaries are able to purchase mortgage default insurance from the government on their portfolio of FRMs[8]. The credit guarantee reduces the risk of banks' assets, allowing intermediaries to increase both their leverage and the risk of their mortgage portfolio. As in the real world, intermediaries can invest in two distinct mortgage contract types: FRMs and ARMs. At origination, the intermediary prices the entire repayment structure of each contract, contingent on the characteristics of mortgages in the model—long-term, prepayable, and defaultable. The intermediary's risk-taking capacity is determined endogenously by the optimal fraction of each mortgage type held in the balance sheet and the quantity of insurance acquired from the government. The severity of the intermediaries' financing constraints will, in turn, endogenously determine the risk premium they charge for default exposure. Ultimately, the financial intermediary computes menus of competitively priced mortgage contracts for FRMs and ARMs.

These menus display how mortgage interest rates respond to variations in the credit worthiness of individual borrowers, as indicated by variables such as loan-to-value (LTV) and payment-to-income (PTI) ratios at origination. Borrowers face idiosyncratic house valuation shocks which influence their optimal default decisions. Individual borrowers assess the market value of their mortgage debt, including mortgage payments. If this value proves excessively large compared to the after-shock housing value, the borrower defaults. Consequently, both high leverage and/or elevated mortgage payments endogenously lead to mortgage default. Within this framework, the specific characteristics of each mortgage contract give rise to distinct default rate paths. Relative to FRMs, ARMs typically have lower required payments during recessions, resulting in less cyclical and lower default rates. Mitigating the cyclicality of default rates for FRMs is possible only when refinancing is frictionless, which is not the case. In an economy without credit guarantees, the convexity of the mortgage interest rate for FRMs, as a function of the borrower's leverage, exceeds that of the ARM contract. This discrepancy arises due to the pro-cyclicality of ARM payments. During downturns, lower ARM payments lead to lower default rates, while larger payments during upswings, when default rates are low, make the ARM contract more profitable[9]. Conversely, in an economy with credit guarantees, the credit risk associated with FRMs is no longer priced, rendering FRM rates insensitive to borrower leverage. The presence of the credit guarantee allows borrowers to increase their leverage with this type of debt without incurring the substantial risk premium observed in a non-subsidized economy. Consequently, this leads to

[8]For a thorough examination of the guarantee fees charged by the GSEs to provide mortgage default insurance, refer to Section 7.3. Additionally, in Section 7.3.1, I show evidence highlighting the distinction in guarantee fees between FRMs and ARMs, supporting the model assumption that insurance is exclusively available for FRMs.

[9]It is worth noting that my model focuses on a real economy, and therefore, I abstract from mortgage contract choice trade-offs related to inflation risk, a topic explored by Campbell and Cocco (2003).

an endogenous shift from an economy where ARMs dominate to one in which FRMs become prevalent—a phenomenon observed in the US.

The comprehensive quantitative model features four distinct agent types: borrowers, savers, financial intermediaries, and the government. Aligned with asset pricing literature, intermediaries issue riskless deposits, engage in investments in risky assets such as mortgages, purchase the credit guarantee offered by the government, and face financial frictions. Impatient borrowers seek to finance a portion of their consumption by borrowing from more patient savers but can only do so through the financial intermediary, pledging their housing as collateral. Therefore, borrowers' consumption will be highly sensitive to the supply and pricing of credit from intermediaries. The government will charge a non-contingent fee, known as the guarantee fee, to the intermediary to cover mortgage defaults. Any excess losses not covered by this guarantee fee will be taxed on borrowers and savers.

While my model incorporates rich heterogeneity and imperfect risk sharing between agents, it will be enough to keep track of the aggregate wealth of borrowers, savers, and intermediaries as state variables[10]. Keeping track of the entire distributions would render my problem intractable. The aggregation result stems from the individual portfolio choice sets scaling linearly in their respective individual wealth. Despite these properties, I can still model financial distortions and introduce a rich set of aggregate shocks. The model economy transitions between a normal state and a housing-crash state characterized by heightened house price uncertainty, aggregate interest rate risk[11], and aggregate business-cycle income risk.

To quantify the impact of eliminating the credit guarantee, I use my model to compute three different economies. In the first economy, I match the guarantee-fee observed in the data and calibrate the model to the US economy—defined as the *baseline economy.* Around 70% of FRMs in the economy come with a credit guarantee, mirroring the data. FRMs constitute the majority of outstanding mortgage debt, and borrowers optimally choose a FRM share of 71.5%[12]. In the first counterfactual economy, I raise the guarantee-fee such that there are no insured FRMs, while permitting the borrower to optimally choose the ARM share—defined as the *optimal ARM share economy.* In this economy, the ARM share amounts to 56.7%. Lastly, in the second counterfactual economy, termed the *constrained ARM share economy*, I not only disallow mortgage insurance (akin to the *optimal ARM share economy*) but also impose a constraint on the FRM share, ensuring

[10] Given the long-term nature of mortgages, I will also need to track the history of outstanding mortgage debt and mortgage payments. In a model without long-term mortgages, tracking only the aggregate wealth values would suffice.

[11] This risk pertains to the interest rate on ARMs, given the absence of an explicit monetary policy rule in the model. Additionally, this shock correlates with the aggregate income shock.

[12] Despite the aggregation result, the coexistence of ARMs and FRMs persists because the default probability of a mortgage type increases with the quantity of that mortgage supplied.

it does not fall below 71.5% (akin to the *baseline economy*). The purpose of this economy is to eliminate the credit guarantee while keeping the mortgage choice from becoming endogenous.

In the *baseline economy*, the regulatory advantages incentivize the issuance of riskier mortgages with high LTV and PTI ratios compared to the counterfactual economies. At origination, the average LTV in this economy is 86%[13], while the PTI stands at 36%. These values closely align with those observed in the US economy. In the *optimal ARM share economy*, the LTV decreases to 81.7%, and the PTI is reduced to 33.2%. With intermediaries now bearing the entire mortgage credit risk, the mortgage portfolio becomes riskier, leading to a subsequent reduction in mortgage leverage. However, in the *constrained ARM share economy* leverage decreases even further. The average LTV is 80.3%, while the PTI drops to 32.6%. This highlights the substantial impact of mortgage choice, as borrowers recognize the pricing benefits of ARMs and are incentivized to increase leverage. The intermediary's balance sheet also undergoes significant changes. For instance, in the *optimal ARM share economy*, the bank's equity is on average 11.3% larger, and during a crisis, it is 15.6% larger relative to the *baseline economy*; these increases are 9.4% and 12.5%, respectively, for the *constrained ARM share economy.* Once again, this result underscores the importance of endogenizing the mortgage choice. When banks optimize, they exhibit better capitalization, especially during downturns, even with slightly larger mortgage leverage. Lastly, this finding suggests that the provision of credit would remain stable even during crisis periods without a credit guarantee.

A larger and riskier mortgage portfolio results in higher mortgage default rates. In the *baseline economy*, the average default rate is 1.8%, and during a crisis, the default rate rises to 3.9%. In contrast, in the *optimal ARM share economy*, default rates decrease to 0.8% on average and almost halve during a crisis, reaching 1.9%. We can isolate the leverage effect from the mortgage choice effect by examining default rates in the *constrained ARM share economy*, which average 0.9% and increase to 2.1% during a crisis. Financial intermediaries experience greater losses during a financial crisis without the endogenous mortgage choice. Ultimately, these losses will impact the stability of borrower consumption and house prices.

One of the objectives of providing a public credit guarantee is to lower mortgage interest rates, thereby enhancing affordability. The model rationalizes this as well. In the *baseline economy*, the mortgage rates on FRMs and ARMs average 3.6% and 3.7%, respectively. Despite high FRM leverage and the large mortgage default rates studied before, FRM rates are lower. The fact that the rate for ARMs is larger than that for

[13]For conforming loans securitized by the GSEs, the median LTV at origination is 80%, while for Ginnie Mae loans, it is around 97%. Considering the market shares of these agencies in the US mortgage market, as illustrated in Figure 11, the median LTV at origination for mortgages with some form of public guarantee is approximately 86%.

FRMs reflects the pricing of the credit guarantee. When I eliminate the credit guarantee on FRMs, both mortgage rates increase, indicating that the benefit of holding insured FRMs spills over to the ARM rate. Specifically, in the *optimal ARM share economy*, the mortgage rate on FRMs increases to 4.4%, while that for ARMs rises to 4.0%. FRMs become more expensive. When I do not allow for mortgage choice to be endogenous, the mortgage rate on FRMs increases even further to 4.6%, while that for ARMs drops to 3.6%. These numbers demonstrate that eliminating the credit guarantee while allowing borrowers to select different mortgage products would not only keep mortgage provision safe but also prevent a substantial increase in mortgage prices.

In terms of welfare, the standard deviation of consumption and welfare is smallest in the *optimal ARM share economy* for both borrowers and savers. Specifically, for borrowers, relative to the *baseline economy*, the standard deviation of consumption drops by 4.2% with an endogenous mortgage choice and by 3.9% when the ARM share remains fixed. A riskier economy generates more volatile leverage and higher default rates, impacting the overall volatility of consumption. Furthermore, when mortgage choice is allowed to be endogenous, the pro-cyclical payments from ARMs provide extra insurance to borrowers.

All of these results rely on an important component of the model: mortgages are long-term contracts. The FRM and ARM portfolios held by the financial intermediary are traded and priced in the secondary market. Mortgage rates priced at origination react to future expected repayment schedules through these secondary market prices. With the credit guarantee, these mortgages can be traded at higher values even during crises (resembling the liquidity of mortgage-backed securities that the government boosts through the GSEs). The market value of the FRM portfolio is 1% lower in the *optimal ARM share economy*, relative to the *baseline economy*, while it is 2.1% lower in the *constrained ARM share economy*, reflecting the slightly higher default rate. For the ARM portfolio, the changes are smaller since the credit guarantee does not directly impact it.

I conduct two additional quantitative exercises in the paper. The first one compares the three economies under a housing recession scenario, characterized by periods of low aggregate income coupled with an increase in the cross-sectional dispersion of house values. I compute impulse response functions to assess the performance of the mortgage and housing markets in each of these economies. I demonstrate that in the *baseline economy*, mortgage risk is too large due to the credit guarantee, resulting in default rates increasing 50% more compared to the other economies. Nevertheless, mortgage rates do not increase as much due to the credit guarantees. Furthermore, financial intermediaries keep issuing high-leverage mortgages, making the convergence to the stationary steady state more sluggish. In the *constrained ARM share economy*, the large share of FRMs in the balance sheet leaves the financial intermediary too exposed to credit risk during downturns. Compared to the *optimal ARM share economy*, default rates and mortgage rates increase by more, while house prices and leverage suffer more significant reductions. The endogenous mortgage choice allows intermediaries to better hedge against default risk.

Finally, I also compute the model-based menus for FRMs and ARMs. I calculate (counterfactual) mortgage rates as a function of the loan-to-value ratio at origination for the *baseline economy* and the *optimal ARM share economy*. In the economy without credit guarantees, for a borrower with an LTV of 75%, the mortgage rate for FRMs is approximately 30 basis points (bps) higher than that for ARMs. This difference expands to 90 bps for highly-leveraged borrowers with LTVs above 90%. In the economy with credit guarantees, FRM rates become insensitive to borrower leverage. For a borrower with an LTV of 75%, the mortgage rate for FRMs is approximately 15 bps lower than that for ARMs, reflecting the low default rates. For highly-leveraged borrowers with LTVs above 90%, the interest rate on ARMs is now around 120 bps higher than that of FRMs, reflecting the fact that the GSEs subsidy eliminates credit risk, especially for riskier borrowers. This also suggests that without credit guarantees, the homeowners who would switch to ARMs are those with high LTVs.

Outline Section 3 outlines the quantitative general equilibrium model. Section 4 discusses the calibration strategy and solution method. Section 5 shows the results obtained from the stationary distributions associated with each of the economies described in this section. I also present the impulse response functions to the housing recession scenario, and the model-based menus of mortgage interest rates. Finally, section 6 provides concluding remarks.

2 Literature Review

This paper contributes to several strands of literature. The existing literature on mortgage design highlights the first-order importance that mortgage characteristics have on both households' welfare and the macroeconomy. The welfare benefits of ex-ante mortgage designs have been studied both in the empirical literature (Fuster and Willen, 2017; Di Maggio et al., 2017; Ganong and Noel, 2020) and the structural quantitative macroeconomic finance literature (Eberly and Krishnamurthy, 2014; Campbell and Cocco, 2015; Campbell et al., 2021; Guren et al., 2021; Piskorski and Tchistyi, 2011; Piskorski and Seru, 2018). Although different in structure, these studies agree that those designs in which mortgage payments are higher in booms and lower in recessions perform better than designs with fixed mortgage payments for risk and insurance reasons, improving consumption and lowering mortgage default. The literature has also found that the ex-post government interventions that are focused on reducing monthly payment significantly reduce default, increase consumption, and even allow house prices to recover faster; the Home Affordable Refinance Program (HARP) (Agarwal et al., 2023) and the Home Affordable Modification Program (HAMP) (Ganong and Noel, 2020) are a few examples. However, the existing literature only compares an economy with all fixed-rate mortgages (FRMs) against one with all-alternative mortgage contract (ARMs, etc.) and therefore cannot speak to the trade-offs that arise with mortgage choice. On the contrary, my paper studies the properties that arise when I endogenize the choice between ARMs and FRMs. Furthermore, I highlight the role that the credit guarantee in the US plays in shaping this endogenous choice.

Furthermore, the quantitative papers in this literature focus on the borrower's decision problem while treating the financial intermediary as a zero-profit condition. My model studies the general equilibrium, by solving a dynamic problem of the financial intermediary. One exception stands out, (Greenwald et al., 2021) studies shared appreciation mortgages (SAMs) that index payments to aggregate house prices in a model with a rich financial sector similar to mine. They find that this mortgage payments indexation reduces financial fragility and improves risk-sharing. However, they do not allow for an endogenous mortgage choice.

I also contribute to the mortgage choice literature that began with Campbell and Cocco (2003), which constructed a stylized life-cycle model with two types of mortgage contracts, FRMs and ARMs, endogenously imposing the (mortgage) interest rates and house price shocks in the model. Later papers developed this literature (Chambers et al., 2009; Garriga and Schlagenhauf, 2010; Corbae and Quintin, 2015); notably, Liu (2022) used UK data to show that high-LTV borrowers, who pay large initial credit spreads, trade off their insurance motive with reducing credit spreads over time using shorter-term contracts. I find similar results when computing the model-based menus. Finally, in (Oosthuizen and Sánchez Sánchez, 2023), which features a mortgage choice, I study how restricted access to teaser-rate mortgages during the Great Recession amplified the housing bust. I argue that the crisis was magnified by constrained buyers and mortgagors holding teaser-rate mortgages who valued greatly the back-loaded payment structure embedded in teaser-rate mortgages. This literature is normally solved in partial equilibrium, focusing on the borrower's problem. My contribution to this literature is to solve the general equilibrium model, by (i) solving a dynamic problem for the financial intermediary who prices both long-term contracts while still allowing for the mortgage choice to be endogenous, and (ii) studying the general equilibrium effects that the government credit guarantees have on mortgage choice.

My paper also contributes to the literature analyzing housing finance reform and the role of the government in mortgage markets. Elenev et al. (2016) focuses mainly on aggregate risk and the role of financial intermediaries. Increasing the price of the mortgage guarantee crowds in private capital, reduces financial fragility, and leads to fewer but safer mortgages. Another nearby strand of literature analyzes the distributional effects of removing the GSE subsidies (Jeske et al., 2013; Gete and Zecchetto, 2018). However, these papers abstract from mortgage choice and are not able to speak to the aggregate or distributional effects of removing the credit guarantee on the housing and mortgage markets when the financial intermediaries are able to substitute the typical 30-year FRM with an alternative mortgage structure. Finally, (Fuster and Vickery, 2015) show that the FRM market share is sharply lower when securitization liquidity is impaired and mortgages are instead generally retained in portfolio, highlighting the relevance of agency-MBSs for maintaining the FRM share. Their results are, however, only studied in partial equilibrium.

3 The Model

To study the consequences on the US mortgage and housing markets of eliminating the government mortgage credit guarantee, I develop a new dynamic general equilibrium model of the mortgage market that emphasizes the role of the financial sector and the government in shaping mortgage availability in the economy. The model is designed to study how mortgage risk is shared among agents in the economy. I set up a model of incomplete risk-sharing between three types of agents: mortgage borrowers (B), savers (S), and financial intermediaries (I). Figure 26 in the Appendix 7.6 depicts the model structure.

3.1 Preferences

All households maximize expected utility. The economy is populated by two types of infinitely-lived households: borrowers (B), and savers (S). Borrowers are less patient than savers ($\beta^S > \beta^B$). Assume both types have the same attitude towards risk, represented by the constant relative risk aversion γ.

Denote by $a \in \{B, S\}$ the agent's type for borrowers and savers, respectively. The utility function depends on non-durable consumption c_t^a, housing consumption s_t^a, and deposit holdings d_t^a as follows:

$$u^a(c_t^a, s_t^a, d_t^a) = \frac{1}{1-\gamma}\Big((c_t^a)^{1-\theta^a}(s_t^a)^{\theta^a}\Big)^{1-\gamma} + \psi^a \frac{(d_t^a)^{1-\gamma}}{1-\gamma}. \tag{1}$$

The utility over non-durable and housing consumption is Cobb-Douglas as in Berger, Guerrieri, Lorenzoni, and Vavra (2018). The preference parameters θ^a and ψ^a can potentially vary across agents. θ^a refers to the share of consumption in housing services. ψ^a is a parameter which generates a demand for holding bank deposits, without explicitly modelling their role as special liquid assets (for example, Diamond (2020)).

3.2 Markets

There are several assets in the economy. The model features competitive markets for housing, riskless bank deposits, mortgages, equity in the financial intermediary firm, and a special market in which households can trade claims to their endowment income only with households of their same type-a.

3.2.1 Endowment Asset

This is an endowment economy. At each period t, borrowers and savers will demand shares (n_t^a) of the aggregate endowment. The aggregate payoff to the endowment asset received by both households is defined Y_t^{mod}. It has two components. The first one is the total economy's output, Y_t. This is the first source of **aggregate risk** in the model. Y_t follows an AR(1) process, defined as the *aggregate income shock*. The second component of Y_t^{mod} equals the losses incurred from mortgage default which are rebated[14]; I describe this element in detail in section 3.9.2. Each period, borrowers receive a fraction v^B of the aggregate endowment Y_t^{mod}, where $0 < v^B < 1$. The remaining fraction $v^S = 1 - v^B$ is paid to the savers.

I make the simplifying assumption that households can trade their endowment ownership only with other type-a agents, the endowment asset trades at price $p_t^{a,n}$. The aggregation results presented in detail in section 3.5.4 crucially depend on the assumption that households are required to trade all their assets[15].

[14] Quantitatively this term is always small, and it is used for tractability.

[15] If an individual household received its own non-tradeable endowment asset, it would be necessary to track the entire distribution of wealth for each type-a agent as in Krusell and Smith (1998), which would make my problem intractable. This assumption on the endowment asset resembles the one made in Diamond and Landvoigt (2021)

3.2.2 Housing

Borrowers and savers can hold housing, a durable asset. One unit of housing offers one unit of housing services that can be consumed by agents. Each period t, there is a fixed stock of housing $\bar{H}$ which can be traded at a price p_t^h. Type-a agents hold h_t^a units of housing[16], such that $\bar{H} = \sum_a h_t^a$. Housing consumption requires a per-unit maintenance payment, equal to $\delta^h \cdot p_t^h$, to stay in use. Making this maintenance payment is mandatory and not a choice. Finally, borrowers (and not savers) face an idiosyncratic shock to the value of their housing. A borrower who, at the end of period t, owns h_t^B units of housing, will face a multiplicative shock ϵ_{t+1} to its housing value at the beginning of period $t+1$, $\epsilon_{t+1} \cdot p_{t+1}^h h_t^B$. The shock ϵ_{t+1} is drawn from a mean one lognormal random variable, i.i.d. across households and across time. The variance of ϵ_{t+1} evolves as a binary Markov chain, which is the second source of **aggregate risk** in the economy. We refer to high realizations of the variance of ϵ_t as *the housing risk shock*.

3.2.3 Deposits

The financial intermediary issues deposits that can be held by both the borrowers (d_t^B) and savers (d_t^S). One unit of deposits held in period t pays the risk-free interest rate r_{t+1}^d in period $t+1$.

3.2.4 Mortgages

Mortgages can only be held by the financial intermediary. This is consistent with the fact that mortgages are held primarily by financial institutions and not directly by households. Borrowers can ask for these loans, against their housing, in the mortgage market. Mortgages are long-term contracts, defaultable, and prepayable. For tractability and similar to Hatchondo and Martinez (2009), mortgages are modeled as perpetuities with outstanding loan balances that decline geometrically. One unit of debt yields payments of $1, \delta, \delta^2, \ldots$ until the borrower prepays or defaults; the fraction $(1-\delta)$ captures the scheduled amortization for the principal. In my model, mortgage contracts are characterized by (i) the mortgage type, (ii) the mortgage debt outstanding, and (iii) the mortgage interest payment. The details are described below.

Prepayment Households prepay their old mortgage debt outstanding in order to refinance. I make several assumptions. First, mortgagors can only refinance into the same mortgage type they held previously, that is

[16] I can alternatively assume separate housing markets, in which each type-a agent trades housing units at a price $p_t^{a,h}$ only among type-a agents, and they are able to hold $\bar{H}^a$ such that $\bar{H} = \sum_a \bar{H}^a$.

if you prepay an old FRM, you cannot refinance into an ARM[17]. Second, the probability of refinancing will be exogenous and not part of the choice set of the borrower[18]. Nevertheless, and crucial to the model, the leverage of the newly originated mortgage is an endogenous choice. Since the model is infinite horizon and mortgages are modelled as perpetuities, the only newly originated mortgages will come from refinancing.

Default Households can also default on their mortgages. Borrowers will hold the two types of mortgages simultaneously. That is, a mortgagor can potentially finance a fraction of its housing with a FRM and the remaining fraction with an ARM. This is a consequence of the aggregation result. Therefore, a borrower can *partially default* on one of the two mortgage balances, or can *completely default* on its total mortgage debt. The detailed wealth expressions will be described in section 3.5.1.

Described in section 3.2.2, households receive an idiosyncratic shock ϵ_{t+1} to their housing value which exogenously guides the default decision of borrowers. However, mortgage leverage and (importantly) mortgage interest payments will also endogenously drive the borrower's default decision. For analytical expressions on how default is optimally chosen (default thresholds) see expressions (93) and (96).

Mortgage Types A first-order defining characteristic of the mortgages in my model is the mortgage type. Specifically, it refers to the type of rate chosen at origination. This can be either a fixed-rate or a variable rate. When originating a mortgage, the financial intermediary offers the borrower two set of menu contracts, one for each mortgage type. The borrower will take as given these two set of contracts, however internalizing the effect that choosing a certain individual portfolio of assets (α^B) will have on the mortgage rates offered. The full definition of α^B is given in section 3.3. The two menus will vary with the state of the economy at origination ($\mathcal{Z}_{orig}$), and potentially on the current state of the economy ($\mathcal{Z}_t$).

FRMs. This menu of contracts is defined as,

$$\iota^{frm}(\alpha^B_\tau, \mathcal{Z}_\tau, \mathcal{Z}_t) \equiv \iota^{frm}(\alpha^B_\tau, \mathcal{Z}_\tau), \quad \forall\, \mathcal{Z}_t \tag{2}$$

This is the interest rate a mortgagor who originates a FRM in period τ, and holds a portfolio α^B_τ, will use to calculate its mortgage payment in period t. Notice that the origination occurs in period τ, and hence the interest rate offered depends on the state of the economy when it was originated, $\mathcal{Z}_\tau$. However, by definition

[17] This is merely a simplifying assumption that can easily be expanded.

[18] Allowing for endogenous refinancing is one of the next steps of this project.

a FRM implies payments that are not contingent on the current state of the economy, $\mathcal{Z}_t$.

ARMs. The total mortgage interest rate on adjustable rate mortgages is defined as,

$$\iota^{arm}(\alpha_\tau^B, \mathcal{Z}_\tau, \mathcal{Z}_t) \equiv \underbrace{spread^{arm}(\alpha_\tau^B, \mathcal{Z}_\tau)}_{\text{Fixed = Priced by bank at } \tau} + \underbrace{i_t(\mathcal{Z}_t)}_{\text{Variable}} . \tag{3}$$

This is the interest rate that a mortgagor who originates an ARM in period τ, and holds a portfolio α_τ^B, will use to calculate its mortgage payment in period t. The interest rate includes two components. The first one is $spread^{arm}(\alpha_\tau^B, \mathcal{Z}_\tau)$, which is the menu of ARM contracts offered by the financial intermediary[19] in period τ. The key aspect of $spread^{arm}(\alpha_\tau^B, \mathcal{Z}_\tau)$ is that it will remain *fixed* for the term of the contract, and therefore will not depend on the state of the economy in any other period. The second component of the ARMs rate is $i_t(\mathcal{Z}_t)$, the policy rate set by an exogenous monetary authority[20]. This is the third source of **aggregate risk** in the economy. i_t will follow an AR(1) process, defined as the *interest rate shock*. This is the way in which ARMs in this model are *variable rate* contracts[21].

Principal and Interest Strips After being originated by banks, mortgages can be traded on secondary markets. Each mortgage vintage has a potentially different origination rate and hence a different secondary market price. However, you can replicate any portfolio of different mortgage vintages using two instruments: an interest-only (IO) and a principal-only (PO) strip, a result similar to the one found in (Greenwald et al., 2020). Let $p_t^{F,I,j}$ and $p_t^{F,m,j}$ be the secondary market prices of the IO and PO strips faced by the financial intermediary, respectively. For details, see the intermediary's problem in section 3.7.

I will also divide the mortgage asset into PO and IO strips for the borrower. Due to aggregation, all assets should be re-traded and re-balanced at the trading stage. Hence, I need separate prices for the *mortgage principal asset* and for the *mortgage payment asset*. In appendix 7.8, I provide a simple example of why we need these two assets. The trading prices are $p_{t+1}^{B,I,j}$ and $p_{t+1}^{B,m,j}$ for the IO and PO strips, respectively[22].

Evolution of Principal Balance and Interest Payments In the model, new originations only come from those agents who were able to refinance in the previous period. See section 3.4 for details on the timing. Old mortgage debt is any debt that was originated in any period before that. In the rest of the paper, I will

[19] Resembles $\iota^{frm}(\alpha_\tau^B, \mathcal{Z}_\tau)$ for the FRMs.

[20] I do not model a Taylor Rule explicitly.

[21] In the real world, the first component is called the *margin*, while the second one is called the *index*.

[22] Notice that these prices are different from the secondary market prices faced by the Financial Intermediary.

use the index $l \in \{nr, r\}$ to describe variables that were chosen by agents that go through the refinancing stage in the previous period; i.e. chosen by refinancers ($l = r$), or variables that were chosen by agents that did not go through the refinancing stage; i.e. chosen by non-refinancers ($l = nr$).

Equations (175) - (176) describe the evolution of the aggregate outstanding mortgage principal balances. For each mortgage type $k \in \{frm, arm\}$, these transition functions depend on the old non-defaulted mortgage balances ($M_t^{nr,k}$) and the non-defaulted originations ($M_t^{r,k}$) from one period before[23].

Equation (177) describes the evolution of aggregate interest payments from FRMs. This transition function depends on the old non-defaulted interest payments from FRMs ($M_t^{I,frm}$), and the mortgage interest payments from newly originated FRMs, which depends on the pricing function described in equation (2).

Finally, equation (178) describes a similar transition function but for ARMs. Equation (178) actually describes the evolution of the spread payments on ARMs coming *uniquely from the fixed part of the total interest payments*, as described in equation (3). It depends on the old non-defaulted spread payments from ARMs ($M_t^{S,arm}$), and the spread payments from newly originated ARMs, which depends on the pricing function described in equation (3). To compute the aggregate amount of interest owed on the *variable part of the total interest payments*, I use the transition function for mortgage balances ($M_t^{nr,arm}$) described in equation (176), since it resets every period and therefore I do not need to keep track of the history of $i_t(\mathcal{Z}_t)$.

Notice that $M_t^{nr,k}$, $M_t^{I,frm}$, and $M_t^{S,arm}$ will be state variables of the model. In Appendix 7.7, I compute simplified transition functions for these variables but without assuming default or refinancing.

3.2.5 Financial Intermediary's Equity

The financial intermediary's equity can only be held by savers. In period t, the savers demand intermediary equity shares b_t^{ξ}. One unit of this assets pays x_{t+1}^{ξ} in period $t+1$, which is a fraction of the financial intermediary's dividend. The exact definition of x_{t+1}^{ξ} is given in section 3.7, as a prior description of the financial intermediary's problem is needed. As with the other assets of the model, in each period the equity asset trades at a price of p_t^{ξ}.

3.3 Portfolio Holdings (notation)

Now, I am ready to define the whole vector of portfolio holdings for borrowers and savers.

[23] For aggregate quantities I will use capital letters, for the individual borrower's problem I will use a lower case version of these variables.

Non-refinancers Define the portfolio at period t of a borrower who does not refinance as $\alpha_t^{B,nr}$. Non-refinancers hold housing (h_t^B), shares of the endowment asset (n_t^B), deposits ($d_t^{B,nr}$), mortgages outstanding at a fixed-rate ($m_t^{nr,frm}$) and at a variable rate ($m_t^{nr,arm}$), mortgage payment due on the FRM ($m_t^{I,frm}$), and the spread payment due on the ARM ($m_t^{S,arm}$). All these assets are purchased/traded by the borrower at the trading stage in period t, as described in 3.4. Then $\alpha_t^{B,nr}$ is defined as,

$$\alpha_t^{B,nr} = \left\{h_t^B, n_t^B, d_t^{B,nr}, m_t^{nr,frm}, m_t^{nr,arm}, m_t^{I,frm}, m_t^{S,arm}\right\} \tag{4}$$

Refinancers The portfolio at period t of a borrower who did refinance is $\alpha_t^{B,r}$. Refinancers hold housing (h_t^B), shares of the endowment asset (n_t^B), deposits which they can re-balance at the refinancing stage ($d_t^{B,r}$), refinanced mortgages outstanding at a fixed-rate ($m_t^{r,frm}$) and at a variable rate ($m_t^{r,arm}$). The housing and endowment assets are purchased/traded by the borrower at the trading stage in period t, as described in 3.4. The re-balanced deposits, and the refinanced mortgages are originated/issued as described in 3.4 under the refinancing stage problem description. Then $\alpha_t^{B,r}$ is defined as,

$$\alpha_t^{B,r} = \left\{h_t^B, n_t^B, d_t^{B,r}, m_t^{r,frm}, m_t^{r,arm}\right\} \tag{5}$$

Savers Define the portfolio at period t of a saver as α_t^S. Savers hold housing (h_t^S), shares of the endowment asset (n_t^S), deposits (d_t^S), and shares of the financial intermediary's equity (b_t^ξ). Then α_t^S is defined as,

$$\alpha_t^S = \left\{h_t^S, n_t^S, d_t^S, b_t^\xi\right\} \tag{6}$$

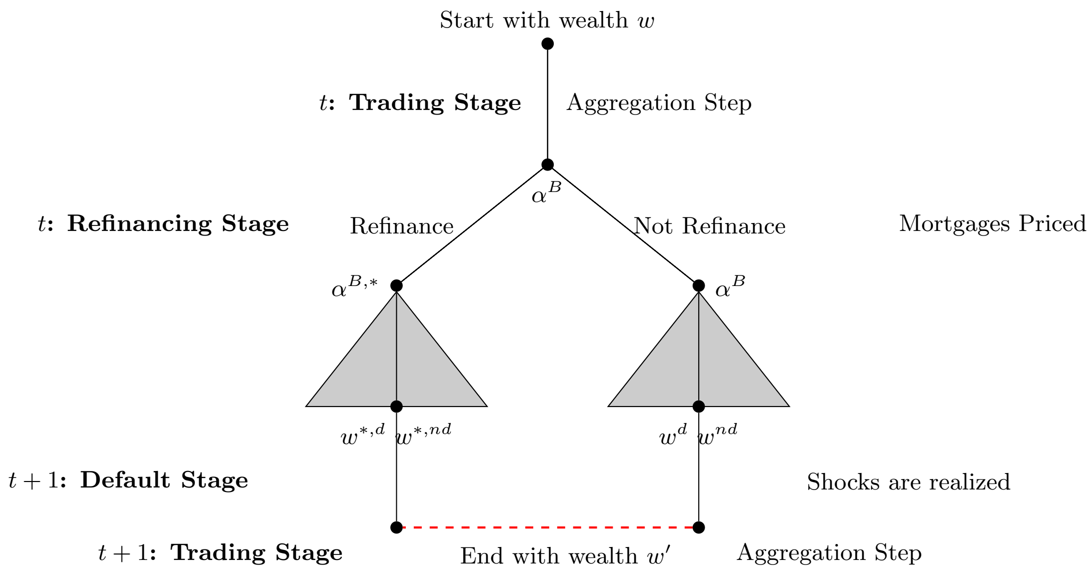
3.4 Timing

In this section, I describe the timing of the borrower's problem. Then I will extend it to the other agents of the model. Figure 1 depicts the timing of the model.

Default Stage: At the beginning of any period, each borrower will enter a stage where they can partially or completely default on their mortgage outstanding balances.

Shocks are realized. For explanatory purposes assume that the borrower is standing at the beginning of period $t+1$. Immediately after the period begins all shocks are realized: income shock Y_{t+1}, idiosyncratic housing risk shocks ϵ_{t+1}, and the policy rate shock i_{t+1}.

Figure 1: Model's Timing Representation



Default occurs. Borrowers are already *heterogeneous* when they enter the default stage. First, because their housing value depends idiosyncratically on their specific housing shock ϵ_{t+1}. Secondly, because at the end of the last period (t) some borrowers were able to refinance (and hence, hold $\alpha_t^{B,r}$) and some were not (and hence, hold $\alpha_t^{B,nr}$). I allow for both refinancers and non-refinancers to default. Conditional on the refinancing status, there are 4 potential *post-default* wealth levels, (i) no-default, (ii) partial default on either the FRM or the ARM, and (iii) total default. The precise wealth expressions can be found in section 3.5.

Trading Stage: The trading stage is the second stage occurring in any period. Borrowers enter with their *post-default and heterogeneous* wealth levels to this stage. It is in this period that I impose the *aggregation result*, described in detail in section 3.5.4. The assumption I make in this stage is that all assets available to the borrower trade in competitive markets[24]. Based on this assumption the whole borrower's choice set will scale linearly with wealth[25]. It is then a consequence that all borrowers will aggregate to a single representative borrower[26]. In this stage, the representative agent will choose the portfolio $\alpha_t^{B,nr}$ described in (4). On top of that, the representative borrower also chooses consumption of non-durables (c_t^B), and housing services consumed (s_t^B). The problem faced by the borrower in this stage is described in section 3.5.

[24]Even though some markets are not accessible to all agents, i.e. mortgages for savers or the intermediary's equity for borrowers.

[25]If borrower 1 chooses the vector $\alpha_t^{B_1}$ using its post-default wealth w_t^1, and borrower 2 enters the trading stage with wealth $w_t^2 = \kappa \cdot w_t^1$, with $\kappa > 0$ then its optimal portfolio will be $\alpha_t^{B_2} = \kappa \cdot \alpha_t^{B_1}$.

[26]This results is similar to the one found in Diamond and Landvoigt (2021), and similar to that paper it is used here for tractability.

Refinancing Stage: Once borrowers choose the portfolio from the trading stage, $\alpha_t^{B,nr}$, a fraction of them will refinance (with some exogenous probability q^r). The fraction of borrowers that refinance will be allowed to not only refinance its mortgage balances but also extract (or save) some of its liquid wealth in the form of deposits[27]. By the aggregation result, there is a single representative agent entering this stage, hence this borrower will (potentially) hold both mortgage types. An assumption I make in this stage is that the borrowers that get to refinance will originate new loans for both mortgages[28],

$$\{d_t^{B,nr}, m_t^{frm,nr}, m_t^{arm,nr}\} \to \{d_t^{B,r}, m_t^{frm,r}, m_t^{arm,r}\},$$

after the refinancing stage, these borrowers will end up with the portfolio $\alpha_t^{B,r}$, as defined in section (5). Those that do not refinance will simply continue holding their *trading stage* portfolio $\alpha_t^{B,nr}$. A key element of this stage is that *mortgage pricing* occurs here. The financial intermediary offers the two sets of menu contracts $\{\iota^{frm}, spread^{arm}\}$ defined in (2) and (3), respectively. Based on these menus the borrower chooses $\alpha_t^{B,r}$, and based on $\alpha_t^{B,r}$ the financial intermediary offers $\{\iota^{frm}, spread^{arm}\}$[29] . A fixed point problem. The complete problem for a refinancer is described in section 3.5.

3.5 Individual household problem

In this section, I delineate the recursive optimization problem for borrowers. I will present it moving 'backwards' through the different stages. I start with the problem borrowers face in the default stage in period $t+1$, continuing with the problem in the refinancing stage at period t, and concluding with the problem borrowers solve at the trading stage in period t. This follows the logic in Figure 1.

Comment on notation. Variables in general will be indexed by several categories, $x_t^{a,k,l,j}$. The only index not yet introduced is $k \in \{d, nd, (d,j)\}$, which refers to the default decision: total default (d), no default (nd), or partial default on mortgage type-j (d,j). a refers to the agent type, l refers to the refinancing status, and j refers to the mortgage type.

Value Function notation. In period t, the value function borrowers face at the trading stage will be denoted as $V^B(w_t^B, \mathcal{Z}_t)$, a function of the *post-default* wealth w_t^B, and the aggregate state of the economy

[27] This will allow me talk about cash-in refinance and cash-out refinance.
[28] This is a simplifying assumption that can easily be expanded and allow for *partial refinancing*.
[29] The problem of the intermediary will be described in section 3.7.

in period t. In that same period t, borrowers entering the refinancing stage will face $V^{B,l}(\cdot)$ depending on whether they refinanced ($l = r$) or not ($l = nr$), the inputs of each of the two refinancing stage functions differ. I will describe them in the current section.

3.5.1 Default Stage Problem

Borrowers choose whether to default on their mortgages, taking as given that they are maximizing a value function $V^B(w_t^B, \mathcal{Z}_t)$ that is increasing in their wealth w_t^B when entering the trading market. All the wealth levels in this section will be conditional on the refinancing state, $l \in \{nr, r\}$[30].

Non-housing wealth The non-housing wealth is composed of the endowment and deposit assets. $y_{t+1}^{B,n}$ is the payoff in period $t+1$ generated from holding one unit of the endowment asset n_t^B. Its specific definition can be found in section 3.9 where I introduce the aggregate endowment income.

$$w_{t+1}^{B,nh,l} = (y_{t+1}^{B,n} + p_{t+1}^{B,n}) \cdot n_t^B + r_{t+1}^d \cdot d_t^{B,l} \tag{7}$$

Notice that as described in section 3.3, deposits will be conditional on the refinancing status while the endowment asset will not be allowed to be modified during the refinancing stage in period t.

Non-defaulters wealth The following expression depends on (i) non-housing wealth (7), (ii) a taxation term T_{t+1}^B, (iii) housing wealth, and (iv) a mortgage term which is comprised of mortgage interest payments and mortgage outstanding balances. Taxes T_{t+1}^B will be used to cover the part of the mortgage losses that go beyond the guarantee-fee[31]. Taxes will be proportional to the endowment shares held in period t, for a full description see equation (53). Terms (iii) and (iv) are what is known as the housing equity.

$$\begin{aligned} w_{t+1}^{B,nd,l}(\epsilon_{t+1}) &= w_{t+1}^{B,nh,l} - T_{t+1}^B + \left(1 - \delta^h\right) p_{t+1}^{B,h} \cdot \epsilon_{t+1} \cdot h_t^B \\ &\quad - \sum_{j=\{arm,frm\}} \left(\frac{\text{payment}_{t+1}^{l,j}}{m_t^{l,j}} \cdot (1 + \delta \cdot p_{t+1}^{B,I,j}) + (1-\delta) + \delta \cdot p_{t+1}^{B,m,j} \right) m_t^{l,j}. \end{aligned} \tag{8}$$

The housing wealth in period $t+1$ equals $\left(1 - \delta^h\right) p_{t+1}^{B,h} \cdot \epsilon_{t+1} \cdot h_t^B$. It includes the mandatory maintenance cost δ^h per unit of housing held, and the idiosyncratic housing shock ϵ_{t+1}.

[30] As in the previous mortgage description section, the equations that I present in this section will be based on the assumption that *there is no partial refinancing*, that means that if an agent refinances its FRM outstanding debt in period t it will also refinance its ARM outstanding debt in period t.

[31] That means they could be negative (a transfer), too.

The last term in (8) is related to the mortgages which are collateralized by housing. It includes both mortgages. The first component of the last term on equation in (8) refers to the IO asset due at period $t+1$, including the resale value at the trading stage. The second component of this term is the principal payment which only depends on the decaying parameter δ, and the final term is the resale value of the PO asset. The interest mortgage payments functions were introduced in section 3.2.4, here I write them compactly. For the FRM, and conditional on the refinancing status, the payments due at period $t+1$ are described by,

$$\text{payment}_{t+1}^{l,frm} = \begin{cases} m_t^{I,frm} & \text{if } l = nr \\ \underbrace{\iota^{frm}\left(\alpha_t^{B,r}, \mathcal{Z}_t\right)}_{\text{Pricing function set at } t} \quad m_t^{r,frm} & \text{if } l = r \end{cases} \tag{9}$$

On the other hand, payments on an ARM due at period $t+1$ are described by,

$$\text{payment}_{t+1}^{l,arm} = \begin{cases} m_t^{S,arm} + i_{t+1} \cdot m_t^{nr,arm} & \text{if } l = nr \\ \Big(\underbrace{spread^{arm}\left(\alpha_t^{B,r}, \mathcal{Z}_t\right)}_{\text{Pricing function set at } t} + \, i_{t+1}\Big) \cdot m_t^{r,arm} & \text{if } l = r \end{cases} \tag{10}$$

Partial defaulter's wealth A borrower is allowed to partially default on one of its two mortgages. Similar to the *non-defaulter's* wealth (8), the following expression depends on (i) non-housing wealth (7), (ii) a taxation term T_{t+1}^B, (iii) (partial) housing wealth, and (iv) a mortgage term which is comprised of payments and mortgages outstanding for the *non-defaulted mortgage*. Terms (iii) and (iv) represent the housing equity of partial defaulters. If the borrower defaults on mortgage type $j \in \{arm, frm\}$, but not on mortgage type $\kappa \in \{arm, frm\}$ $(j \neq \kappa)$ its wealth at the default stage in period $t+1$ will be,

$$\begin{aligned} w_{t+1}^{B,(d,j),l}(\epsilon_{t+1}) = (1-\lambda^j) \cdot w_{t+1}^{B,nh,l} - T_{t+1}^B - \left(1-\delta^h\right) p_{t+1}^{B,h} \cdot \epsilon_{t+1} \cdot h_t^B \cdot \chi_t^{l,\kappa} \\ - \left(\frac{\text{payment}_{t+1}^{l,\kappa}}{m_t^{l,\kappa}} \cdot (1+\delta \cdot p_{t+1}^{B,I,\kappa}) + (1-\delta^\kappa) + \delta^\kappa \cdot p_{t+1}^{B,m,\kappa}\right) m_t^{l,\kappa}. \end{aligned} \tag{11}$$

The partial defaulter loses a fraction of its housing and is only allowed to keep a fraction $(1-\lambda^j)$ of its non-housing wealth[32]. The variable $\chi_t^{l,\kappa}$ represents the *fraction of housing that is funded by the mortgage*

[32] Although in the US mortgages in general have no recourse, this cost could reflect lower FICO scores, monetary costs of default, etc.

type that was not defaulted, κ,

$$\chi_t^{l,\kappa} = \frac{m_t^{l,\kappa}}{m_t^{l,\kappa} + m_t^{l,j}} \tag{12}$$

notice that this applies to both refinancers and non-refinancers. The partial defaulter is then allowed to keep $\chi_t^{l,\kappa}$ of its housing in exchange for not defaulting on the mortgage balance $m_t^{l,\kappa}$ for one more period (at least) and keeping up with the mortgage interest payments, $\text{payment}_{t+1}^{l,\kappa}$. Finally, notice that all borrowers will pay the same tax amount, I assume $(1-\lambda^j)$ will not multiply the taxes T_{t+1}^B.

Total defaulter's wealth Finally, the borrower can decide to default on its whole mortgage debt, i.e. both mortgages. In that case, the borrower's wealth is,

$$w_{t+1}^{B,d,l} = (1-\lambda^{arm}-\lambda^{frm}) \cdot w_{t+1}^{B,nh,l} - T_{t+1}^B, \tag{13}$$

Therefore, if the borrower defaults completely, it loses all of the housing it owns and is allowed to keep only a fraction $(1-\lambda^{arm}-\lambda^{frm})$ of its deposits and endowment assets.

3.5.2 Refinancing Stage Problem

A household has the option to prepay and refinance its mortgage. A borrower enters the refinancing stage with the portfolio α_t^{nr}, as described in equation (4). Refinancing occurs at the end of each period, for notation purposes I will assume that the borrower is standing at the end of period t (see Figure 1).

Budget Constraint A refinancing household needs to prepay its mortgage *face values*[33], $m_t^{nr,frm} + m_t^{nr,arm}$. Denote the liquid funds $(f_t^{B,r})$, after prepayment, as

$$f_t^{B,r} \equiv d_t^{B,nr} - \sum_{j=\{arm,frm\}} m_t^{nr,j} \tag{14}$$

I will allow the refinancer to use part of its deposits (not its endowment asset, though) to fund part of the new originations (cash-out refinancing) or to save an extra amount (cash-in refinancing) after the new

[33] As I did in mortgage description section, the equations that I present in this section will be based on the assumption that *there is no partial refinancing*, that means that if an agent refinances its FRM outstanding debt in period t it will also refinance its ARM outstanding debt in period t. This can easily be extended.

originations occur. The budget constraint faced by borrowers who refinance is,

$$f_t^{B,r} \geq d_t^{B,r} - \sum_{j=\{arm,frm\}} \left[m_t^{r,j} - m_t^{r,j} \cdot C^{refi} \left(\frac{m_t^{r,j}}{h_t^B} \right) \right] \tag{15}$$

where $m_t^{r,j}$ for $j \in \{frm, arm\}$ are newly originated (refinanced) outstanding mortgage balances. Refinancing is costly, the borrower is required to pay $C^{refi}(\cdot)$ per unit of dollars lent (for each mortgage-type j). Finally, $d_t^{B,r}$ are the new deposits demanded by the borrower who refinances. Therefore, at the end of period t, the refinancers will hold a new portfolio $\alpha_t^{B,r}$ as described in equation (5).

Pricing Functions In this stage, the financial intermediary will 'price' the new originations. It will offer two menu of contracts, one for each mortgage-type,

$$\left\{ \iota^{frm} \left(\alpha_t^{B,r}, \mathcal{Z}_t \right), spread^{arm} \left(\alpha_t^{B,r}, \mathcal{Z}_t \right) \right\} \tag{16}$$

The borrower will not be able to modify such functions, but will however internalize the effect of choosing $\alpha_t^{B,r}$ on the specific interest rate for the FRM and spread for the ARM offered by the intermediary. I will refer to these marginal effects as the *mortgage pricing function derivatives*, which are required to compute the effective returns on all assets. For the FRMs these are defined as,

$$\frac{\partial \iota^{frm} \left(\alpha_t^{B,r}, \mathcal{Z}_t \right)}{\partial a} \tag{17}$$

for any asset $a \in \alpha_t^{B,r}$. For the ARMs these are defined as,

$$\frac{\partial spread^{arm} \left(\alpha_t^{B,r}, \mathcal{Z}_t \right)}{\partial a} \tag{18}$$

for any asset $a \in \alpha_t^{B,r}$. The first order conditions derived in 7.12 require analytical expressions for these functions in order to be able to fully characterize the borrower's portfolio problem. I compute the mortgage pricing function derivatives in section 7.19.

Value Functions Define the value function of the borrower at the refinancing stage as $V^{B,r}(f_t^{B,r}, n_t^B, h_t^B, \mathcal{Z}_t)$, where the liquid resources available to the borrower $f_t^{B,r}$ were defined in (14). Then the problem of a borrower who refinances can be written as,

$$V^{B,r}(f_t^{B,r}, n_t^B, h_t^B, \mathcal{Z}_t) = \max_{\alpha_t^{B,r}} E_t \Bigg[max \Big\{ V^B \left(w_{t+1}^{B,d,r}, \mathcal{Z}_{t+1} \right), V^B \left(w_{t+1}^{B,(d,arm),r}, \mathcal{Z}_{t+1} \right), \\ V^B \left(w_{t+1}^{B,(d,frm),r}, \mathcal{Z}_{t+1} \right), V^B \left(w_{t+1}^{B,nd,r}, \mathcal{Z}_{t+1} \right) \Big\} \Bigg] \tag{19}$$

subject to the budget constraint at the refinancing stage (15), and the pricing functions (16). $V^B(\cdot)$ is the value function faced by the borrower at the trading stage, and defined in section 3.5.3. In particular, it is evaluated at the trading stage in period $t+1$, after the default stage in period $t+1$ has already passed.

The value function for those borrowers who do not refinance is defined as $V^{B,nr}(\alpha_t^{B,nr}, \mathcal{Z}_t)$. Notice that it depends on the whole portfolio vector chosen at the trading stage in period t,

$$V^{B,nr}(\alpha_t^{B,nr}, \mathcal{Z}_t) = E_t \Big[\max \Big\{ V^B \left(w_{t+1}^{B,d,nr}, \mathcal{Z}_{t+1} \right), V^B \left(w_{t+1}^{B,(d,arm),nr}, \mathcal{Z}_{t+1} \right), \\ V^B \left(w_{t+1}^{B,(d,frm),nr}, \mathcal{Z}_{t+1} \right), V^B \left(w_{t+1}^{B,nd,nr}, \mathcal{Z}_{t+1} \right) \Big\} \Big], \tag{20}$$

these borrowers simply carry forward their portfolio from the trading stage.

3.5.3 Trading Stage Problem

In the trading stage borrowers trade *all* their assets among *all* borrowers. This is the key stage for aggregation. *Heterogeneous* borrowers enter this stage in period t after the refinancing stage in period $t-1$, and the default stage in period t. The state variable for each individual borrower is their wealth level[34] after those two stages (w_t^B) and the aggregate state of the economy ($\mathcal{Z}_t$). At each trading stage, borrowers will choose the portfolio vector $\alpha_t^{B,nr}$ described in (4), consumption c_t^B, and housing services s_t^B.

For convenience, define the total value of the mortgage portfolio for the representative borrower in period t at the beginning of the trading stage,

$$\bar{m}_t^B = p_t^{B,m,frm} m_t^{nr,frm} + p_t^{B,m,arm} m_t^{nr,arm} + p_t^{B,I,frm} m_t^{I,frm} + p_t^{B,I,arm} m_t^{S,arm} \tag{21}$$

this portfolio is composed of all the (non-defaulted) mortgage vintages originated until period $t-1$, they satisfy the transition functions for mortgage principal balances and mortgage interest payments, see section

[34] I do not have to *individually* keep track of mortgage balances or mortgage payments precisely because there exists a trading stage in which all the assets in the borrower's portfolio can be traded in competitive markets. This is not true in the aggregate where I do in fact keep track of them.

7.26 .

The full problem of a borrower household is,

$$V^B(w_t^B, \mathcal{Z}_t) = \max_{c_t^B, s_t^B, \alpha_t^{B,nr}} \frac{\left((c_t^B)^{1-\theta^B}(s_t^B)^{\theta^B}\right)^{1-\gamma}}{1-\gamma} + \frac{\psi^B \cdot \left(d_t^{B,nr}\right)^{1-\gamma}}{1-\gamma}$$
$$+ \beta \left[q^{nr} \cdot V^{B,nr}(\alpha_t^{B,nr}, \mathcal{Z}_t) + q^r \cdot V^{B,r}(f_t^{B,r}, n_t^B, h_t^B, \mathcal{Z}_t)\right], \tag{22}$$

subject to the budget constraint at the trading stage,

$$w_t^B \geq c_t^B + \rho_t^{B,h} s_t^B + (p_t^h - \rho_t^{B,h}) h_t^B + p_t^{B,n} n_t^B + d_t^{B,nr} - \bar{m}_t^B \tag{23}$$

The household's budget constraint given in equation (23) shows how a household can allocate its wealth w_t^B in the post-default stage of the period. In addition to its wealth, the household receives the total value of its mortgage portfolio $\bar{m}_t^B$ in equation (21), as well as income from renting its housing available h_t^B at price $\rho_t^{B,h}$[35]. The borrower's household can allocate all these resources to obtain non-durable consumption c_t^B, to buy housing at price p_t^h, rent housing services s_t^B at rental rate $\rho_t^{B,h}$, invest in the borrower-specific assets n_t^B at price $p_t^{B,n}$, and invest in deposits $d_t^{B,nr}$.

3.5.4 Borrower's Problem Aggregation

In this section I will characterize the borrower's problem. I will argue that the households actually behave like a representative agent[36]. The key properties that the model needs to satisfy for this to be true are:

1. The budget set need to satisfy the following property: the choice vector $\left\{c_t^B, s_t^B, \alpha_t^{B,nr}, \alpha_t^{B,r}\right\}$ satisfies the budget set with wealth level $w_t^B \iff$
for some constant $k > 0$ the choice vector $\left\{k \cdot c_t^B, k \cdot s_t^B, k \cdot \alpha_t^{B,nr}, k \cdot \alpha_t^{B,r}\right\}$ satisfies the budget set with wealth level $k \cdot w_t^B$. I show that this is true for my model in equations (84) and (85) in the

[35] A comment on the housing asset is in order. Holding h_t^B units of housing does not necessarily translate into h_t^B housing services for the borrower (i.e the term in the utility function). Instead the borrower is able to rent its entire housing holdings at price $\rho_t^{B,h}$, and rent s_t^B units which do enter the utility function directly. Firstly, this way of modelling housing is necessary for the aggregation results to work. Details can be found in the Appendix 7.9, where I characterize the solution of the borrower problem. Secondly, $\rho_t^{B,h}$ is indexed by B, which means I am assuming segmented rental markets (across borrowers and savers). This is not a crucial assumption and I can assume a single rental market (with a unique price ρ_t^h), in which the borrower is able to rent some housing from the saver each period (or vice-versa).

[36] The main idea behind the aggregation result is to show that if some household $B1$ enters the trading stage with post-default wealth w_t^{B1} such that its optimal choice vector is $\{c_t^{B1}, s_t^{B1}, \alpha_t^{B1,nr}, \alpha_t^{B1,r}\}$,then for any other household $B2$ who enters the trading stage with post-default wealth w_t^{B2} such that $w_t^{B2} = k \cdot w_t^{B1}$, then its optimal choice vector will exactly equal $\{k \cdot c_t^{B1}, k \cdot s_t^{B1}, k \cdot \alpha_t^{B1,nr}, k \cdot \alpha_t^{B1,r}\}$.

Appendix (7.9)[37].

2. All the realizations of households' post-default wealth levels in period $t+1$ are linear functions in the household's portfolio choice vector $\left\{\alpha_t^{B,nr}, \alpha_t^{B,r}\right\}$. I show that this is true for my model in equations (79), (82), and (83) in the Appendix (7.9).

3. The utility function is homogeneous of degree $1-\gamma$. This is given by assumption.

The details of the proof can be found in the Appendix (7.9). Proposition 2 proves the aggregation result itself, Proposition 1 proves some characterization results discussed below, required for Proposition 2 to hold. What are the key assumptions required so that these properties are in fact satisfied? For the budget set to be scalable linearly in wealth, I need the individual borrowers to trade all their assets in a competitive market[38]. Second, for all households' post-default wealth to be linear in the asset's portfolio the key assumption is making the idiosyncratic housing shocks ϵ_{t+1} multiplicative, as I show in equations (79), (82), and (83) .

Consequences of the Aggregation Result In this section I show some useful results that come out of the aggregation results shown in the previous section that will help me solve this model's economy. The first result is to show that the value function for any individual borrower has the following functional form,

$$V^B(w_t^B, \mathcal{Z}_t) = v^B(\mathcal{Z}_t)\frac{(w_t^B)^{1-\gamma}}{1-\gamma},$$

I leave the detailed proof for such a statement in Proposition 1 in Appendix (7.9). But the proof tightly connects to the aggregation proof discussed before. Since the portfolio choices scale linearly in the post-default wealth of the borrower, the objective function is homogeneous of degree $1-\gamma$ in its choice vector, and the utility function is also homogeneous of degree $1-\gamma$, then the value function must be homogeneous of degree $1-\gamma$ in its wealth at time t. Finally $v^B(\mathcal{Z}_t)$ is a function on the aggregate state uniquely, and the exact expression can be found at the end of Proposition 1 in Appendix (7.9). An immediate consequence of the functional form result is that I can recast the Bellman equations at the refinancing (19), (20) and trading (22) stages. For convenience, I just show the Bellman equation at the trading stage (22). This can

[37] This property is only satisfied if the pricing functions studied in (16) are homogeneous functions of degree zero with respect to the choice vector $\alpha_t^{B,r}$, I also prove that this is true in the Appendix 7.20 where I dig deeper into the financial intermediary's problem.

[38] Failure to do so would imply solving a model as in Krusell and Smith (1998), where it would be necessary to track the entire distribution of borrower's wealth, which is an infinite dimensional object making this model intractable.

be re-written as follows,

$$
\begin{aligned}
V^B(w_t^B, \mathcal{Z}_t) = & \max_{\{c_t^B, s_t^B, \alpha_t^{B,nr}, \alpha_t^{B,r}\}} \frac{\left(\left(c_t^B\right)^{\theta^B} \left(s_t^B\right)^{1-\theta^B}\right)^{1-\gamma}}{1-\gamma} + \frac{\psi^B \cdot (d_t^{B,nr})^{1-\gamma}}{1-\gamma} \\
& + \sum_{l=\{r,nr\}} \beta^B \cdot q^l \cdot E_t\left[\frac{v^B(\mathcal{Z}_{t+1})}{1-\gamma} \cdot \max\left(\left(w_{t+1}^{B,d,l}\right)^{1-\gamma}, \left(w_{t+1}^{B,(d,arm),l}\right)^{1-\gamma}, \right.\right. \\
& \left.\left. \left(w_{t+1}^{B,d,l}\right)^{1-\gamma}, \left(w_{t+1}^{B,(d,frm),l}\right)^{1-\gamma} \right)\right]
\end{aligned} \tag{24}
$$

subject to the constraints at the trading (23) and refinancing stages (15), and the pricing functions (16).

Default Thresholds Another consequence of the aggregation result is that I am able to compute (analytical expressions for) default thresholds. Only households that receive a sufficiently bad idiosyncratic housing shock ϵ_{t+1} will default on their mortgage. All borrowers choose mortgages with identical ex-ante risk at time t (due to aggregation), therefore households choose to default if and only if their default wealth is higher than the no-default wealth. See Bellman equation (24).

The refinancing status in period t will impact default in period $t+1$, hence the default thresholds will be conditional on $l \in \{r, nr\}$. Furthermore, borrowers can partially default on either the FRM or the ARM. Solving the model requires solving four default threshold values: $\hat{\epsilon}_{t+1}^{l,k}$, for each combination of l and k. To exemplify, $\hat{\epsilon}_{t+1}^{r,arm}$ is the *ARM default threshold for borrowers who refinanced*: any borrower who went through the refinancing stage in period t that draws a value of ϵ_{t+1} below $\hat{\epsilon}_{t+1}^{r,arm}$ will default on its ARM debt.

Next I show two results. First, I show analytical expressions for the default thresholds with some accompanying intuition. Second, I will compare the ARM's default threshold to that of the FRM.

Default Threshold Expression In appendix 7.11, I show the derivations for the default thresholds. The value for $\hat{\epsilon}_t^{l,arm}$ equals,

$$
\begin{aligned}
& \hat{\epsilon}_{t+1}^{l,arm} \cdot \underbrace{\left(1-\delta^h\right) p_{t+1}^{B,h} \cdot h_t^B \cdot \chi_t^{l,arm}}_{\text{Housing value funded with an ARM}} \\
& = \underbrace{\left(\frac{\text{payment}_{t+1}^{l,arm}}{m_t^{l,arm}} \cdot (1 + \delta \cdot p_{t+1}^{B,I,arm}) + (1-\delta) + \delta \cdot p_{t+1}^{B,m,arm}\right) m_t^{l,arm}}_{\text{Mortgage Market Value = Cost of not defaulting on the ARM}} - \underbrace{\lambda^{B,arm} w_{t+1}^{B,nh,l}}_{\text{Cost of defaulting on the ARM}}
\end{aligned} \tag{25}
$$

Expression (25) explains how the model endogenously generates default on ARMs. If the market value of the ARMs debt is large, this will increase $\hat{\epsilon}_{t+1}^{l,arm}$, which makes sense since the debt burden is higher. Notice that the structure of the mortgage payments are an important part of this term. On the other hand, if the fraction of housing funded with ARMs is large or the non-housing wealth is large (which is partially lost after default), the value of $\hat{\epsilon}_{t+1}^{l,arm}$, default becomes costly since defaulting decreases the value of the wealth entering into the trading stage which is used for non-durable consumption and housing service purchases[39].

Default Threshold Comparison The value of $\hat{\epsilon}_{t+1}^{l,frm}$ is similar to that of (25). The expression can be found in equation (96) in the appendix. Standing in period t at the trading stage, for any state z_{t+1} at the default stage, the individual borrower needs to take care of two cases[40]:

1. $\hat{\epsilon}_{t+1}^{l,arm} < \hat{\epsilon}_{t+1}^{l,frm}$, these are states in which there is more default coming from FRMs.

2. $\hat{\epsilon}_{t+1}^{l,arm} > \hat{\epsilon}_{t+1}^{l,frm}$, these are states in which there is more default coming from ARMs.

My model's main goal is to compare between the two mortgage types; it is interesting to ask for which states z_{t+1} the borrower falls under case 1, for example. The inequality $\hat{\epsilon}_t^{l,arm} < \hat{\epsilon}_t^{l,frm}$ can be written as,

$$\frac{mmv_{t+1}^{l,arm} - \lambda^{arm} w_t^{B,nh,l}}{(1-\delta^h)\, p_{t+1}^{B,h} \cdot h_t^B \cdot \chi_t^{arm,l}} < \frac{mmv_{t+1}^{l,frm} - \lambda^{frm} w_t^{B,nh,l}}{(1-\delta^h)\, p_{t+1}^{B,h} \cdot h_t^B \cdot \chi_t^{frm,l}}$$
$$\Rightarrow \frac{mmv_{t+1}^{l,arm} - \lambda^{arm} w_t^{B,nh,l}}{\chi_t^{arm,l}} < \frac{mmv_{t+1}^{l,frm} - \lambda^{frm} w_t^{B,nh,l}}{\chi_t^{frm,l}}$$

For convenience, I defined the mortgage market value, for $l \in \{r, nr\}$ and $j \in \{frm, arm\}$, as

$$mmv_{t+1}^{l,j} = \left(\frac{\text{payment}_{t+1}^{l,j}}{m_t^{l,j}} \cdot (1 + \delta \cdot p_{t+1}^{B,I,j}) + (1-\delta) + \delta \cdot p_{t+1}^{B,m,j} \right) m_t^{l,j}. \tag{26}$$

Some interesting takeaways are:

1. The default rate for ARMs is lower than the one for FRMs if the market mortgage value $mmv_{t+1}^{l,arm}$ for the ARM is lower than the market mortgage value $mmv_{t+1}^{l,frm}$ for the FRM. Mortgage payments play a key role here.

2. Leverage also matters. If the borrower leverages up in one of the mortgages, the default threshold will endogenously increase. This result will prove important in the quantitative exercise, since it will allow me to obtain interior solutions for the mortgage outstanding and origination shares.

[39]The expression for FRMs is analogous.
[40]In appendix 7.10, I show more intuition on these cases.

3. The refinancing status matters. The model is capable of generating scenarios in which refinancers default more on their ARM debt, while non-refinancers default more on their FRM debt. Mortgage payments also play a key role here. This relevant feature of the model can only be generated with long-term mortgages.

3.6 Saver's Problem

In this section, I describe and solve the representative saver's problem[41].

3.6.1 Timing and Definition

The representative saver maximizes its lifetime expected utility given in equation (1) depending on consumption c_t^S, housing services s_t^S, and deposit holdings d_t^S. Holding some wealth level w_t^S, they choose a portfolio composed of housing h_t^S, shares of their endowment asset n_t^S, deposits d_t^S, and shares of the financial intermediary's equity b_t^ξ, as defined in section (3.3). It is worth noting that b_t^ξ is an asset to which only the saver has access. The whole portfolio is defined as α_t^S. To align the borrower and saver problems, and using my description of the model's timing in section (3.4), I assume that savers choose the set $\{c_t^S, s_t^S, \alpha_t^S\}$ at the trading stage. Given that they do not hold mortgages by assumption, the refinancing stage in period t and default stage in period $t+1$ are irrelevant. They move to the trading stage in period $t+1$ and re-optimize.

3.6.2 Saver's Bellman Equation

The state variables for the saver's problem are the wealth level w_t^S, and the aggregate state of the economy $\mathcal{Z}_t$. The saver's Bellman equation can be written as,

$$V^S(w_t^S, \mathcal{Z}_t) = \max_{\{c_t^S, s_t^S, \alpha_t^S\}} \frac{\left((c_t^s)^{\theta^S} \left(s_t^S\right)^{1-\theta^S}\right)^{1-\gamma}}{1-\gamma} + \frac{\psi^S \cdot (d_t^S)^{1-\gamma}}{1-\gamma} + \beta^S E_t\left[V^S(w_{t+1}^S, \mathcal{Z}_{t+1})\right] \tag{27}$$

subject to the budget constraint at the trading stage,

$$w_t^S = c_t^S + \rho_t^{S,h} s_t^S + (p_t^h - \rho_t^{S,h}) h_t^S + p^{S,n} n_t^S + p_t^\xi b_t^\xi + d_t^S \tag{28}$$

[41] Although the savers are able to hold housing, they do not suffer the idiosyncratic risk housing shock (ϵ) as borrowers. As a result, savers immediately aggregate to a single representative agent.

The household's budget constraint (28) shows how a saver can allocate its wealth w_t^S. In addition to its wealth, the saver receives income from renting its housing h_t^S at price $\rho_t^{S,h}$. The saver can allocate all these resources to obtain non-durable consumption c_t^S, to buy housing at price p_t^h, rent housing services s_t^S at rental rate $\rho_t^{S,h}$, invest in the saver-specific endowment assets n_t^s at price $p_t^{S,n}$, hold deposits d_t^S, and invest in the financial intermediary's equity asset b_t^{ξ}[42]. Finally, the wealth of the saver in the following period w_{t+1}^S can be written as,

$$w_{t+1}^S = \left(1 - \delta^h\right) p_{t+1}^h \cdot h_t^S + \left(y_{t+1}^S + p_{t+1}^{S,n}\right) n_t^S + \left(p_{t+1}^{\xi} + x_{t+1}^{\xi}\right) b_t^{\xi} + (1 + r_{t+1}^d) d_t^S \tag{29}$$

For the endowment saver's specific asset, $y_{t+1}^{S,n}$ is the payoff in period $t+1$ generated from holding one unit of the endowment asset n_t^S, while $p_{t+1}^{S,n}$ is the per unit resale price. The specific definition of $y_{t+1}^{S,n}$ can be found in section 3.9.2 where I introduce the aggregate endowment income. The intermediary's equity trades at a price p_{t+1}^{ξ} in period $t+1$, while the cash flow equals x_{t+1}^{ξ} per unit of equity asset b_t^{ξ}, the definition of x_{t+1}^{ξ} can be found in equation (33) where I introduce the intermediary's problem. All the other assets are analogous to that of the borrower.

Saver's Problem Characterization I describe how to solve in detail the saver's problem in the Appendix 7.13. In Appendix 7.14 I compute the first order conditions of the saver's problem.

3.7 Financial Intermediary

The financial intermediary is a profit maximizing firm in a competitive financial market which is owned in equilibrium by the saver. It makes mortgages and issues deposits and equities backed by these mortgages with the goal of maximizing its value to the owners (savers). As described in section 3.5.4, all individual borrowers choose equally risky mortgages, hence the intermediary's balance sheet entering time $t+1$ can be characterized by the values of the fixed-rate mortgages and the adjustable rate mortgages it makes to the representative borrower, and by the payment due on its deposits that it issued in the previous period. Additionally, the financial intermediary is able to pick the fraction of FRMs that will be government-insured. This feature is similar to the real-world process when a bank decides to securitize mortgages through GSEs. Buying such insurance hedges the financial intermediary against default risk, but not against the prepayment

[42] As I explained in the borrower's section 3.5.3, $\rho_t^{S,h}$ is indexed by S, which means I am assuming segmented rental markets (across borrowers and savers). This is not a crucial assumption and I can assume a single rental market (with a unique price ρ_t^h), in which the borrower is able to rent some housing from the saver in each period (or vice-versa).

and interest rate risks that come with originating a FRM. Finally, the intermediary has to satisfy a capital constraint that represents the intermediary's risk-weighted capital requirement faced by banks in real life.

3.7.1 Timing and Definition

I describe in detail the decisions taken by the financial intermediary in each of the model's stages as described in section 3.4 and Figure 1.

Trading Stage The financial intermediary enters each trading stage with some equity, e_t, the endogenous state variable required to solve the bank's problem. At the beginning of each trading stage the financial intermediary holds PO and IO strips of both mortgages which they resell in the secondary market, as introduced initially at the end of section 3.2.4. For mortgage type-k, I define the PO strips as $M^{F,nr,k}$, which the intermediary can trade in period t at prices $p_t^{F,m,k}$. Define the IO strips for FRMs as $M^{F,I,k}$ and the IO strips for ARMs as $M_t^{F,S,arm}$, which the interemdiary can trade at prices $p_t^{F,I,k}$, respectively[43]. These assets will trace the history of past mortgage originations and mortgage interest rates (i.e different mortgage vintages). The market value of the mortgage assets at the beginning of the trading stage in period t equals,

$$\bar{M}_t^F = p_t^{F,m,frm} M_t^{F,nr,frm} + p_t^{F,m,arm} M_t^{F,nr,arm} + p_t^{F,I,frm} M_t^{F,I,frm} + p_t^{F,I,arm} M_t^{F,S,arm}. \tag{30}$$

The bank is required to pay a fraction τ of its equity e_t as dividend payment to the owners,

$$\tau \cdot e_t, \tag{31}$$

the fraction τ will be a model's parameter and the precise definition of the intermediary's equity e_t will be developed later in this section. The final expression can be found in equation (43). The intermediary can also raise additional funding I_t from its owners at a cost of,

$$C^F(I_t) = \chi^F \cdot I_t^2 \tag{32}$$

[43] This notation resemble that introduced for the borrower. The superscript F will denote the variables coming from the Financial Intermediary's problem (whereas in the borrower's problem I used B). Furthermore, for the allocation variables coming from the financial intermediary's problem I use capital letters that represent *aggregate* quantities (for example, M), since it is enough to use the solution of the **representative borrower** to solve the financial intermediary's problem.

where the constant $\chi^F > 0$. If the intermediary's owners obtain I_t of additional funds, the intermediary only receives $I_t - C^F(I_t)$ of newly issued equity. Notice that the *net dividend payment* to the owners equals $\tau \cdot e_t - I_t$. This equals the per unit payoff of the equity asset b_t^ξ held by the saver, x_t^ξ (see section 3.6.2).

$$x_t^\xi = \tau \cdot e_t - I_t. \tag{33}$$

Refinancing Stage New mortgage originations occur for both mortgage types. To refinance, mortgages need to be prepaid by borrowers at face value. The refinancing borrowers can also adjust their deposit holdings after prepayment. For the fraction q^r of borrowers that refinance,

$$\{M_t^{F,nr,frm}, M_t^{F,nr,arm}, D_t^{nr}\} \to \{M_t^{F,r,frm}, M_t^{F,r,arm}, D_t^r\}.$$

Mortgage Pricing New originations provided by the intermediary are priced competitively in this stage, so that the intermediary makes zero economic profits from each loan. The present value of cash flows paid by the borrower (discounted with the intermediary's pricing kernel) determines the menu of contracts, for each mortgage type, as introduced in equation (16).

Mortgage Credit Insurance The intermediary chooses a fraction s_t of the FRMs in its balance sheet that it optimally insures against credit risk. I make several assumptions. (i) To avoid having to keep track of an additional state variable, I model the credit guarantees as one period default insurance, that is, in each period the government is able to choose a *different* fraction s_t. Alternatively I could keep track of the history of the fraction of mortgages that were insured each previous period $\{\ldots, s_{t-2}, s_{t-1}, s_t\}$ which would add another state variable to the model. (ii) The financial intermediary can only guarantee FRMs. In Appendix 7.3.1 I argue that this is a reasonable assumption for the US mortgage market. (iii) The per-period guarantee applies contemporaneously to newly originated mortgages and old mortgage debt. By choosing the fraction s_t the intermediary is insuring the *entire balance sheet of FRMs* it holds at the end of period t. Alternatively I could keep track of the amount of insurance bought by the financial intermediary in the previous periods, but that would add another state variable to the model. In appendix 7.3 I show that this is a relatively inoffensive assumption since the ongoing guarantee-fee is the most relevant part of the total guarantee-fee payment, see Figure 12. (iv) Insured mortgages that are defaulted exit the balance sheet. The assumption is that the government will pay back the intermediary the face value of the mortgage plus the interest payments owed in that period upon default. If I allowed the defaulted mortgages to remain in the balance sheet, that would add an extra state variable. Finally, (v) a government-guaranteed bond is a security with the same duration (maturity and cash-flow structure) as the non-insured mortgages. The only difference is that it carries no mortgage credit risk because of the government guarantee. Alternatively I could split the FRM asset in two, one that keeps track of non-insured FRMs and one that keeps track of insured FRMs.

The government sets the price of insurance per unit of bond to the government, called the guarantee fee and expressed in this paper as ϕ^{fee}. In this model this will be an important policy parameter.

3.7.2 Decisions taken at the Trading/Refinancing Stages

The financial intermediary's problem is only affected by aggregate quantities, I do not need to split the trading and refinancing stage problems. Given equity capital e_t and the aggregate state $\mathcal{Z}_t$, the intermediary chooses: (i) its IO and PO positions in the secondary market for both ARMs and FRMs, $M_t^{F,nr,frm}, M_t^{F,nr,arm}, M_t^{F,I,frm}, M_t^{F,S,arm}$, (ii) new originations, $M_t^{F,r,frm}, M_t^{F,r,arm}$, (iii) insured fraction of FRMs, s_t, (iv) aggregate short-term deposits, D_t[44], and (v) equity issuance, I_t.

I can define the choice variable set for the financial intermediary as α_t^F,

$$\alpha_t^F = \left\{M_t^{F,r,frm}, M_t^{F,r,arm}, M_t^{F,nr,frm}, M_t^{F,nr,arm}, M_t^{F,I,frm}, M_t^{F,S,arm}, s_t, D_t, I_t\right\} \tag{34}$$

Budget Constraint at the Trading/Refinancing Stages

$$\begin{aligned}(1-\tau)e_t + \underbrace{I_t - C^F(I_t)}_{\text{equity issuance is costly}} + D_t + \underbrace{q^r \cdot \sum_{j\in\{frm,arm\}} M_t^{F,nr,j}}_{\text{Prepayments from refinancers}} \\ = \underbrace{\bar{M}_t^F}_{\text{Old Mortgages}} + \underbrace{q^r \cdot \sum_{j\in\{frm,arm\}} M_t^{F,r,j}}_{\text{New Originations}} + C^{orig}\left(\sum_{j\in\{frm,arm\}} M_t^{F,r,j}\right)\end{aligned} \tag{35}$$

On the left hand side, we have the inside equity $(1-\tau)e_t$ remaining after the mandatory dividend (33) is paid, the funds $I_t - C^F(I_t)$ obtained by raising additional equity from the intermediary's owners, the funds raised in the deposit's market (at a promised rate r_{t+1}^d in period $t+1$), and the funds coming from the refinancers

[44] In section 3.9.4, where I describe the market clearing conditions, I show that the deposits are coming from savers, borrowers who refinance, and borrowers who do not refinance.

who are required to prepay their old mortgage face values. These are used to fund the purchase of their old mortgage portfolio in the secondary market (the definition of $\bar{M}_t^F$ can be found in (30)), and the new mortgage originations for those who are able to refinance. Finally, at the time of origination, intermediaries pay a processing cost $C^{orig}(\cdot)$ that is proportional to the mortgage size originated in period t.

Default Stage Due to the aggregation, all individual borrowers choose equally risky mortgages and we can compute default thresholds (see equations (93) and (96)) which imply aggregate default rates for this economy. Given these results we can compute IO mortgage payoff functions (section 3.7.3) and PO mortgage payoff functions (section 3.7.4) at the default stage in time $t+1$. The value of the intermediary's asset will depend on the aggregate state $\mathcal{Z}_{t+1}$, and therefore equity tomorrow will be conditioned by the performance of these assets in each of tomorrow's states, as shown in equation (43).

3.7.3 IO Payoff Functions

ARMs The *per dollar* IO payoff function $\mathcal{IO}^{l,arm}$ is defined as[45],

$$\mathcal{IO}^{l,arm}(\mathcal{Z}_{t+1}, m_t^{l,arm}) = \left(1 - F_\epsilon(\hat{\epsilon}_{t+1}^{l,arm})\right)\left(\frac{\text{payment}_{t+1}^{l,arm}}{m_t^{l,arm}}\right)(1 + \delta \cdot p_{t+1}^{F,I,arm}) + F_\epsilon(\hat{\epsilon}_{t+1}^{l,arm}) \cdot 0 \tag{36}$$

Equation (36) shows the ARM payoff function for the *interest-only* conditional on the refinancing state l, where I use the ARM interest payment function previously defined in (10). The first term represents the fraction of borrowers who end up repaying their debt times the value of that repayment per dollar lent in period t. The first component of the repayment term refers to the cash flow that is entitled to the holder of the IO asset per unit of dollars[46], and the second term represents the resale value at the following trading stage. The second term always equals zero since mortgage interest payments are foregone under default.

FRMs The *per dollar* IO payoff function $\mathcal{IO}^{l,frm}$ is defined as,

$$\begin{aligned}\mathcal{IO}^{l,frm}(\mathcal{Z}_{t+1}, m_t^{l,frm}) = (1 - s_t)\Bigg[&\left(1 - F_\epsilon(\hat{\epsilon}_{t+1}^{l,frm})\right)\left(\frac{\text{payment}_{t+1}^{l,frm}(1 + \delta \cdot p_{t+1}^{F,I,frm})}{m_t^{l,frm}}\right) + F_\epsilon(\hat{\epsilon}_{t+1}^{l,frm}) \cdot 0\Bigg] \\ + s_t\Bigg[&\left(1 - F_\epsilon(\hat{\epsilon}_{t+1}^{l,frm})\right)\left(\frac{\text{payment}_{t+1}^{l,frm}(1 + \delta \cdot p_{t+1}^{F,I,frm})}{m_t^{l,frm}}\right) + F_\epsilon(\hat{\epsilon}_{t+1}^{l,frm})\left(\frac{\text{payment}_{t+1}^{l,frm}}{m_t^{l,frm}}\right)\Bigg]\end{aligned} \tag{37}$$

[45] Consider a log-normal cumulative distribution function $F_\epsilon : \mathbb{E} \mapsto [0,1]$, with a respective probability distribution function $f_\epsilon : \mathbb{E} \mapsto \mathbb{R}$, where $\mathbb{E}$ is the domain of idiosyncratic housing shock $\epsilon_{t+1} \in \{0, \infty\}$.

[46] I can find the variable interest rate inside the $\text{payment}_{t+1}^{l,arm}$ function.

Equation (37) shows the FRM payoff function for the *interest-only* asset conditional on the refinancing state l, where I use the FRM interest payment function previously defined in (9). This payoff function differs from (36) since FRMs can be insured. The terms multiplying $(1 - s_t)$ represent the non-insured part of the FRMs balance sheet, and it replicate the ARM's payoff function expression. The terms multiplying s_t represent the insured part of the FRMs balance sheet. The first component represents the fraction of borrowers that end up repaying their debt times the value of that repayment per dollar lent in period t. The big difference comes from the second term, which represents those insured mortgages that were defaulted. In this case the financial intermediary only receives the mortgage payment but not the resale value, since these assets exit the balance sheet.

3.7.4 PO Payoff Functions

I leave the expressions for the *principal only payoff functions* for the appendix, see 7.15. Equation 133 shows the PO payoff function ($\mathcal{PO}^{l,arm}$) for ARMs, while 134 shows the PO payoff function ($\mathcal{PO}^{l,frm}$) for FRMs. These are similar to the ones studied for the IO assets. The big difference is that upon default, the government pays back the face value of those FRMs that are insured to the holder of the FRM's PO asset. That term shows up in the PO payoff function for FRMs, while the interest payment upon default shows up in the IO assets for FRMs, equation (37). When defaulted, the mortgage exits the balance sheet.

3.7.5 Aggregate Payoffs

Once computed, the PO and IO payoff functions can be used to determine the *aggregate payoffs*. These are required to compute the intermediary's equity (43).

FRMs The *aggregate* payoff function for FRMs, conditional on the refinancing status, is,

$$\mathcal{P}^{l,frm}_{agg}(\mathcal{Z}_{t+1}) = \Bigg(\mathcal{IO}^{l,frm}(\mathcal{Z}_{t+1}, m^{l,frm}_t) + \mathcal{PO}^{l,frm}(\mathcal{Z}_{t+1})\Bigg) \cdot M^{F,l,frm}_t \tag{38}$$

ARMs The *aggregate* payoff function for ARMs, conditional on the refinancing status, is,

$$\mathcal{P}^{l,arm}_{agg}(\mathcal{Z}_{t+1}) = \Bigg(\mathcal{IO}^{l,arm}(\mathcal{Z}_{t+1}, m^{l,arm}_t) + \mathcal{PO}^{l,arm}(\mathcal{Z}_{t+1})\Bigg) \cdot M^{F,l,arm}_t \tag{39}$$

3.7.6 Intermediary's Balance Sheet

I describe the elements of the intermediary's equity in period $t+1$, after the default stage in period $t+1$ has passed, e_{t+1}. This is the equity to which the intermediary will have access in order to optimally choose its portfolio in $t+1$, see equation (34). That is, the equity with which it will enter the trading stage at period $t+1$. It will represent the market value of all the intermediary's assets and liabilities. It will be composed of three elements: (i) mortgage market value for the intermediary, (ii) deposits promised payment, and (iii) the cost of the mortgage insurance provided by the government.

Aggregate mortgage market value The total market value in period $t+1$ of its mortgage holdings is defined as $\mathcal{P}_{agg}(\mathcal{Z}_{t+1})$. It uses the definitions (38) and (39),

$$\mathcal{P}_{agg}(\mathcal{Z}_{t+1}) \equiv \sum_{s=\{nr,r\}} q^r \sum_{j=\{arm,frm\}} \mathcal{P}^{s,j}_{agg}(\mathcal{Z}_{t+1}) \tag{40}$$

Notice that $\mathcal{P}_{agg}(\mathcal{Z}_{t+1})$ depends on the aggregate state, $\mathcal{Z}_{t+1}$.

Deposit's payment The financial intermediary promises to pay a risk-free amount on its deposit liabilities issued in period t at interest rate r^d_{t+1}, $D_t \cdot (1+r^d_{t+1})$. Notice that this payment does not depend on the aggregate state, $\mathcal{Z}_{t+1}$.

Intermediary's insurance payment The final component is the payment (insurance premium) that the financial intermediary needs to make to the government in order to claim the full mortgage insurance.

$$\mathcal{P}^{\text{insurance}} \equiv \phi^{fee} \cdot s_t \cdot \left(\underbrace{q^r \sum_{j\in\{frm,arm\}} M^{r,frm}_t}_{\text{new originations}} + \underbrace{\bar{M}^{F,frm}_t}_{\text{old mortgages}} \right) \tag{41}$$

Importantly, a very relevant feature of $\mathcal{P}^{\text{insurance}}$ is that it **cannot** be conditional on future states $\mathcal{Z}_{t+1}$. In period t, the financial intermediary decides on the fraction of FRMs to be insured, s_t. This includes the new FRMs originations and the market value of old FRMs holdings traded in the secondary markets, $\bar{M}^{F,frm}_t$,

$$\bar{M}^{F,frm}_t = p^{F,m,frm}_t M^{F,nr,frm}_t + p^{F,I,frm}_t M^{F,I,frm}_t \tag{42}$$

Notice $\bar{M}_t^{F,frm}$ is a subset of $\bar{M}_t^F$ defined in (30). The timing structure of the model does not allow me to include $\mathcal{P}^{\text{insurance}}$ in the intermediary's budget constraint[47] (as one might have thought more naturally). Therefore, I analogously include this term in the transition function for the intermediary's equity, e_{t+1}, as a wealth loss that the intermediary suffers in period $t+1$ regardless of the state z_{t+1}.

Bank's Equity Equation The formal definition of the financial intermediary's equity (which implicitly defines the intermediary's balance sheet) at the start of the trading stage in period $t+1$ is,

$$e_{t+1}(\mathcal{Z}_{t+1}) = \mathcal{P}_{agg}(\mathcal{Z}_{t+1}) - \mathcal{P}^{\text{insurance}} - D_t \cdot (1 + r^d_{t+1}) \tag{43}$$

3.7.7 Capital Constraint

My model does not feature the bank's default. However, it faces a risk-weighted capital requirement. In particular, I will define 3 different kinds of assets, each with different risks and hence different weights on the capital constraint: ARMs, uninsured FRMs, and insured FRMs.

ARMs. For this asset we use the components of the balance sheet related to the ARM defined in (39),

$$\mathcal{P}^{arm}_{agg}(\mathcal{Z}_{t+1}) = \sum_{s=\{nr,r\}} q^r \cdot \mathcal{P}^{s,arm}_{agg}(\mathcal{Z}_{t+1}) \tag{44}$$

The parameter $\bar{e}^{arm}$ will determine the minimum equity capital for ARMs required by regulation.

Uninsured FRMs. For this asset I extract the components related to the uninsured FRMs from equation (38), those terms multiplied by $(1 - s_t)$ in the per dollar payoff equations (37) and (134). For more detail on the exact expression, see Appendix 7.16.

$$\mathcal{P}^{frm,uninsured}_{agg}(\mathcal{Z}_{t+1}) = \sum_{s=\{nr,r\}} q^r \cdot \mathcal{P}^{s,frm,uninsured}_{agg}(\mathcal{Z}_{t+1}) \tag{45}$$

$\bar{e}^{frm,uninsured}$ denotes the minimum equity capital for uninsured FRM debt required by regulation.

Insured FRMs. For this asset I extract the components related to the insured FRMs from equation (38), those terms multiplied by s_t in the per dollar payoff equations (37) and (134). For more detail on the exact

[47] This is a consequence of the borrower's aggregation assumption.

expression, see Appendix 7.16.

$$\mathcal{P}_{agg}^{frm,insured}(\mathcal{Z}_{t+1}) = \sum_{s=\{nr,r\}} q^r \cdot \mathcal{P}_{agg}^{s,frm,insured}(\mathcal{Z}_{t+1}) \tag{46}$$

$\bar{e}^{frm,insured}$ denotes the minimum equity capital for insured FRMs debt required by regulation.

Capital Constraint Expression. Given these definitions, the financial intermediary has to satisfy the following set of constraints at time t

$$\begin{aligned} e(\mathcal{Z}_{t+1}) \geq \bar{e}^{frm,insured} \cdot \mathcal{P}_{agg}^{frm,insured}(\mathcal{Z}_{t+1}) + \bar{e}^{frm,uninsured} \cdot \mathcal{P}_{agg}^{frm,uninsured}(\mathcal{Z}_{t+1}) \\ + \bar{e}^{arm} \cdot \mathcal{P}_{agg}^{arm}(\mathcal{Z}_{t+1}), \quad \forall z_{t+1}|\mathcal{Z}_t \end{aligned} \tag{47}$$

In Appendix 7.17 I show that it suffices to impose the constraint for the worst possible aggregate state in $t+1$. This regulatory capital constraint is only occasionally binding. If the intermediary is sufficiently well capitalized, it has a precautionary incentive to save because of the risk of the constraint binding in the future. I view this constraint as reflecting solvency concerns faced by a bank holding a diversified portfolio of securities, with special attention on the benefits of securitizing mortgages through the GSEs.

3.7.8 Financial intermediary's Bellman Equation

The financial intermediary's problem is to optimally choose the optimal portfolio α_t^F (see (34)) in order to maximize the dividend to the owners (net of equity issuance) presented in (33). The savers are the owners of the intermediary's equity, therefore I use the stochastic discount factor of savers[48] to discount the financial intermediary's future cash flows. The full-dynamic problem of the intermediary's can be written as,

$$V^I(e_t, \mathcal{Z}_t) = \max_{\alpha_t^F} \tau e_t - I_t + \mathrm{E}_t \left[\mathcal{S}_{t,t+1}^S \cdot V^I\left(e_{t+1}, \mathcal{Z}_{t+1}\right) \right] \tag{48}$$

subject to the budget constraint (35), the definition of the bank's equity in period $t+1$ (43), and the capital requirement constraint (143). The first order condition of this problem can be found in Appendix (7.18).

[48]The expression for the saver's stochastic discount factor, $\mathcal{S}_{t+1}^S$, can be found in Appendix (128) and as a consequence of solving problem (27) in the main text.

3.8 Government

In the model I consolidate the role of the GSEs and Ginnie Mae, and that of the Treasury Department into one single government[49]. The government is required to make a payment in period $t+1$ to cover the insured losses from the financial intermediary. The government covers the mortgage payments due at the moment of default and prepays the principal owed to the financial intermediary. I define the total government disbursement as Gov_{t+1}^{exp}, in Appendix 7.21 I show in detail how to compute this total expenditure. Notice that this is an expenditure that depends on the state z_{t+1}. On the other hand, the government receives the payoffs from the mortgage premiums, which I already defined in equation (41). A key fact about this payment is that $\text{Gov}_{t+1}^{income}$ *does not depend on period* $t+1$, since the insurance contract was set ex-ante in period t,

$$\text{Gov}_{t+1}^{income} = \mathcal{P}^{\text{insurance}} \tag{49}$$

The difference between the income from the guarantee fee (49) and the expenditure from mortgage defaults (169) will be a cost borne by the taxpayers, which in my model includes both borrowers and savers. Therefore to balance the budget, the government charges taxes that depend on the state z_{t+1}, T_{t+1}. For *every state* z_{t+1}, the (forward-looking) government budget constraint can then be described as[50]

$$T_{t+1} = \text{Gov}_{t+1}^{exp} - \text{Gov}_{t+1}^{income} \tag{50}$$

A functional form for the total taxation levied by the government can be found in section 3.9.3.

3.9 Aggregation, and Model Equilibrium

I define the equilibrium of the economy. But first, I review some definitions to close the model completely[51].

[49] For Ginnie Mae this is immediate, since it is a wholly owned government agency (see section 7.2.2). On the other hand, the GSEs are federally chartered corporations privately owned by shareholders. However, even before the Great Recession the financial housing sector viewed their securities as de facto beneficiaries of the federal government, which turned explicit and a reality as of September 2008.

[50] I assume there is not government debt since that would add an extra state variable.

[51] Uppercase letters will denote aggregate choice variables for savers and borrowers throughout. $M_t^{l,j}$ are the aggregate mortgage face values for $l \in \{r, nr\}$ and $j \in \{frm, arm\}$. $M_t^{I,frm}$ is the aggregate mortgage interest payment due on the FRMs. $M_t^{S,arm}$ is the aggregate mortgage spread payment due on the ARMs. H_t^a is the aggregate housing ownership by agent type $a \in \{B, S\}$. $D_t^{B,l}$ are the aggregate deposit holdings for the borrowers, which are conditioned of the refinancing status. D_t^S are the aggregate deposit holdings for the savers. N_t^a are the aggregate endowment shares holdings by agent type. B_t^{ξ} are the saver's aggregate holdings of intermediary equity. C_t^a is the aggregate non-durable consumption by agent type, and S_t^a is the aggregate housing service consumption.

3.9.1 Endowment Income

The aggregate payoff of all households' endowment is Y_t^{mod}, equal to total output Y_t plus some quantitatively small losses incurred during mortgage default and additional transfers which I will detail in this section. The specific (pre-tax) payoffs received by borrowers and savers from the endowment asset, per unit held, equals

$$y_t^{B,n} = \nu \cdot Y_t^{mod}$$
$$y_t^{S,n} = (1 - \nu) \cdot Y_t^{mod}$$

3.9.2 Aggregate Endowment Income

The total payoff Y_t^{mod} is described as,

$$Y_t^{mod} = Y_t + Y_t^{rebate} \tag{51}$$

The second term Y_t^{rebate} allows me to maintain the simplicity of an endowment economy, while satisfying the resource constraint (see equation (64)). The first term incorporated to Y_t^{rebate} represents the non-housing wealth losses from those households that defaulted, $Y_t^{rebate,B}$. The second term incorporated to Y_t^{rebate} represents the total (housing) resources lost and borne by the financial intermediary, $Y_t^{rebate,I}$. I detail the expressions for these two elements in Appendix 7.22. The total rebate is simply

$$Y_t^{rebate} = Y_t^{rebate,B} + Y_t^{rebate,I} \tag{52}$$

3.9.3 Taxation

The tax revenue from the borrowers and savers comes from endowment income taxation, the tax rate paid by both agents in period $t+1$ equals τ_{t+1}. The taxes paid by the borrower in period $t+1$, T_{t+1}^B,

$$T_{t+1}^B = \tau_{t+1} \cdot \nu \cdot Y_{t+1} \cdot n_t^B \tag{53}$$

For the case of savers, the equation is analogous,

$$T_{t+1}^S = \tau_{t+1} \cdot (1 - \nu) \cdot Y_{t+1} \cdot n_t^S \tag{54}$$

The total taxes levied by the government to cover the default losses not covered by the guarantee-fee (see equation (50)) are defined as

$$T_{t+1} = T^{B}_{t+1} + T^{S}_{t+1} \tag{55}$$

3.9.4 Market Clearing Equations

This section states the market-clearing conditions necessary to define an equilibrium of this economy (section 7.23). The market-clearing condition for mortgage debt requires that intermediaries purchase the entire portfolio of mortgages (for $l \in \{r, nr\}$ and $j \in \{frm, arm\}$)

$$M^{l,j}_{t} = M^{F,l,j}_{t} \tag{56}$$

The market-clearing condition for the entire portfolio of mortgage interest payments due on FRMs,

$$M^{I,frm}_{t} = M^{F,I,frm}_{t} \tag{57}$$

The market-clearing condition for the entire portfolio of mortgage spread payments due on ARMs,

$$M^{S,arm}_{t} = M^{F,S,arm}_{t} \tag{58}$$

The market-clearing condition for the housing capital,

$$H^{B}_{t} + H^{S}_{t} = \bar{H} \tag{59}$$

The market-clearing condition for the rental markets for each $a \in \{B, S\}$,

$$S^{a}_{t} = H^{a}_{t} \tag{60}$$

The market-clearing condition for deposits,

$$\begin{aligned} D^{B}_{t} &= q^{nr}_{t} \cdot D^{B,nr}_{t} + q^{r}_{t} \cdot D^{B,r}_{t} \\ D_{t} &= D^{B}_{t} + D^{S}_{t} \end{aligned} \tag{61}$$

The market-clearing condition for the intermediary equity,

$$B_t^{\xi} = 1 \tag{62}$$

The market-clearing condition for the endowment asset for each $a\{B, S\}$,

$$N_t^a = 1 \tag{63}$$

The market-clearing condition for the non-durable consumption good,

$$Y_t = C_t^B + C_t^S + C^F(I_t) + C^{orig}\left(\sum_{j\in\{frm,arm\}} M_t^{F,r,j}\right) + \sum_{j\in\{frm,arm\}} M_t^{r,j} \cdot C^{refi}\left(\frac{M_t^{r,j}}{H_t^B}\right) + \delta^h \cdot p_t^h \cdot \bar{H} \tag{64}$$

The total endowment equals total consumption, the cost from mortgage origination and equity issuance from the intermediary, the refinancing cost from the borrower's problem, and the mandatory housing depreciation cost. The detailed equilibrium definition can be found in the Appendix 7.23.

4 Calibration, and Solution Method

I calibrate the model to annual U.S. data. I choose to calibrate the model to the most recent values to which I have access. The objective is to compare the current state of the US to a counterfactual economy in which the GSEs' credit guarantee is eliminated or is so expensive that the bank decides not to pay for it.

4.1 Calibration

Table 1 lists each of the parameters in the model. The calibration is based on the economy with $\phi^{fee} = 0.0063$, with this parameter value the model generates a mean for the insured FRMs of 71.5%. This value closely matches the fraction of Residential Mortgages funded through Agency-MBS, see Figure 10.

Preferences. Risk aversion is set to a standard value of 1, implying log utility. A choice of $\gamma = 1$ or $\gamma = 2$ is chosen in several macro-housing and intermediary asset pricing models. I choose the saver discount factor β^S to match the deposit rate in the model to the annualized real yield of 1-year treasury bills from 1980 until 2019, which is 1.3%. I choose this time span because it was around the mid-80s that the GSEs start becoming relevant for the US mortgage funding market, I do not consider the Covid period. The mean value for the deposits interest rate in the model is 1.7%. The borrower's discount factor governs their appetite for mortgage debt, and so it can be set to target the household debt to GDP ratio in the data. In the model, mortgage debt to GDP is 43%, while using the Quarterly Report on Household Debt and Credit dataset from the Federal Reserve and the NIPA tables the same ratio equals 49% using data from 1999 until 2019. I set the saver's housing weight θ^S on the Cobb-Douglas preferences[52] to match the housing wealth to GDP ratio observed in the data. Using the Financial Accounts of the United States at the Federal Reserve[53] and NIPA, the average housing wealth to GDP ratio from 1986-2019 equals 1.3. The model estimates a slightly larger average of 1.9. Finally, ψ is chosen to match the liquidity premium estimated in (Krishnamurthy and Vissing-Jorgensen, 2012) which is around 73 basis points, the model estimates a liquidity premium of 91 basis points for the savers.

Financial Intermediary I set the equity capital requirements for the financial intermediary based on the Basel regulatory requirements for mortgage assets. The risk-weighted under Basel II and III, on first liens on a single-family home that are "prudently underwritten and performing" enjoy a 50% risk weight and all others a 100% risk weight. Agency MBS receive a 20% risk weight[54]. I take the conservative route and assume that all mortgages that "stay" in the balance sheet of the intermediary will be assigned a 100% risk weight or an 8% capital requirement; these apply to both FRMs and ARMs. For the mortgages securitized through the GSEs, I assign the 20% risk weight or an 1.6% capital requirement, see also (Elenev et al., 2016). I assume a quadratic functional form for the origination cost function, $C^{orig}(x) = \frac{\mu^{orig}}{2} \cdot x^2$, of the intermediary. I calibrate the parameter μ^{orig} to match the mortgage spread on FRMs with respect to the deposits rate to the analogous data spread between the 30-year FRMs and the 10-year treasury bill, the mean value in the data from 1986 to 2019 is 1.7 the model produces an average spread of 1.8%. For the remaining parameters I follow (Elenev et al., 2021), in their paper they compute time series of dividends, and equity issuance for all publicly-traded banks in the US. Over the period from 1974 to 2018, banks paid out an average 6.8% of their book equity per year as dividends, there is set τ to that exact value. The net payout rate for the same banks (i.e., net of equity issuance) is estimated to be around 5.7% as a fraction of the bank's equity, which my model closely matches with an equity issuance parameter of around $\chi = 1000$.

[52]I set the borrower's housing weight θ^B equal to that of savers. Althought the model allows for this felxibility to exist.

[53]I use the balance sheet of households and nonprofit organizations, therefore I exclude the corporate sector from the calculations.

[54]Ginnie Mae receives a 0% risk weight given the explicit guarantee.

Housing There is a fixed supply of housing, which I assume equal to 1. For the value of δ^h, I match the depreciation of residential fixed assets taken from the BEA which on average since 1982 equals 2.3% of residential fixed assets, this will be value for the forced maintenance rate of housing in the model. The idiosyncratic house price dispersion shock (ϵ_{t+1}) follows a two-state Markov Chain, the low value represents normal times while the high value represents an inflated housing risk environment. The probability of remaining in the normal state the next period is 95% while the probability of remaining in the high housing risk state next period is 80%. The erdogic distribution reveals that under these parameters the economy spends 80% of the time in the normal state. The transition probabilities are chosen to match 2 moments regarding the probability and duration of financial crisis. I am interested in housing recessions, in this paper I define housing recessions as events in which not only housing risk increases but it comes combined with a drop in the aggregate endowment. (Jordà et al., 2016) find that most financial crises after WW-II are related to the mortgage market, with losses mounting to 1/5 of annual real GDP over 5 years[55]. I pick the probability of remaining in the high housing risk state next period (80%) to match the average duration of housing recessions (5 years), and pick the probability of remaining in the low housing risk state next period (95%) to match the unconditional probability of a housing recessions (around 5%).

What follows is to choose the specific value for the dispersion of the housing risk shock. I define σ_ϵ^{low} as the low dispersion shock value (normal times), and σ_ϵ^{high} as the high dispersion shock value (high housing risk times) to match the average default rates on residential mortgages during normal times and during housing recessions. The average default rate[56] for the GSEs-backed mortgages rose from 0.5% in 2007 to around 4% for Freddie Mac and 5.5% for Fannie Mae in 2009 (Goodman et al., 2023). Both default rates came down slowly for a decade to reach their pre-crisis average of 0.5% in 2019. For the covid crisis, these default rates increased to 3% for both GSEs. The current averages sit at 0.5% again. For the mortgages backed by Ginnie Mae the story is a little different. Serious delinquency rates for FHA loans in 2007 was around 4.5%, it rose to 10% in 2009, to slowly drop to 4% in 2019. During the covid crisis, this average default rate reached 11%. During the Great Recession, Ginnie Mae's mortgage origination was around a fifth of the total of mortgage originations backed by either one of the GSEs or Ginnie Mae (Goodman et al., 2023), this number (one fifth) is similar when calculating the outstanding agency-MBS, see Figure 11. Therefore, in the model I will target a default rate of 1% - 1.5% during normal times, and a default rate of 5% - 6%

[55] Worse than normal crisis, i.e. crisis where the financial sector is not affected.

[56] In my paper I define default rate as serious delinquency which are those mortgages that have 90 days or more past due or in the foreclosure process.

during housing recessions. The model generates an average default rate in normal times of 1.8%, and an average default rate of 4% during housing crisis. To achieve the default rate during normal times I set the value of $\sigma_{\epsilon}^{low} = 0.198$. On the other hand, to achieve the default rates during housing crisis I set the value of $\sigma_{\epsilon}^{high} = 0.252$. Finally, the unconditional severity rate parameter faced by the financial intermediaries (ζ) are assumed to be 25%, in line with the values estimated in the literature, for instance (Campbell et al., 2011) estimate that value to be 28%.

Mortgages The average duration of a 30-year FRM is about seven years. This low duration is mostly the result of early prepayments. The parameter δ captures amortization absent refinancing or default. A yearly value of $\delta = 0.97$ implies that 3% of principal is paid-off in the first year of the mortgage. Assuming an interest rate of 5.20%, which is the average mortgage rate on 30-year FRMs from 2000-2020 the amortization on the first year is around 1.5%[57]. The refinancing rate is calibrated to match the observed refinancing rate in the data, (Greenwald, 2018) estimates that the average refinancing rate for Fannie Mae 30-Year MBS (FNM30) is 17.8% over the sample 1994 - 2015. I set the refinancing probability equal to 18%. I have to pin down two different loan-to-value ratios. The first one is related to the new mortgage originations, and the other is related to the mortgage balances of those borrowers that have held their mortgages for longer periods (in the model, the non-refinancers). The refinancing cost function is displayed here,

$$C^{refi}\left(\frac{m_t^r}{h_t^B}\right) = \frac{\phi^{refi}}{2} \cdot \left(\frac{m_t^r}{h_t^B}\right)^2$$

I choose the value of ϕ^{refi} to pin down the average loan-to-value ratio for new originations[58]. The median LTV at origination is 80% for the conforming loans securitized by the GSEs, while for the Ginnie Mae loans it is around 97%. Taking the shares for each of these agencies in the US mortgage market, as observed in Figure 10, the median LTV at origination that comes with some form of public guarantee is around 86%. I choose a value of $\phi^{refi} = 0.02$ which returns an average loan-to-value ratio in the model equal to 86%. On the other hand, I choose the pecuniary default penalty λ to match the average mortgage leverage (for all the borrowers) observed in the SCF, see (Diamond and Landvoigt, 2021). There they estimate that the average loan-to-value ratio in the data is around 64.2%. I choose $\lambda = 0.01$[59] and obtain an average loan-to-value

[57] In reality ARMs normally come with a teaser-rate the amortization is slower in the initial years, which would require $\delta^{frm} < \delta^{arm}$. The model allows for this flexibility but I assume the same value in this version.

[58] Although my model does not accept endogenous refinancing, I can certainly choose the optimal loan size.

[59] I choose the same value for λ whether the borrower defaults on its ARM than it when it defaults on its FRM.

ratio of 67%[60].

Endowment Using annual data on real per capita GDP growth from the BEA NIPA tables from 1929-2019. The standard deviation and autocorrelation of the cyclical HP-filtered series are 0.027 and 0.45, respectively. I convert the continuous AR(1) productivity process to a 3-state Markov chain using the Rouwenhorst method. In the model I set the standard deviation of this AR(1) process equal to 0.023 while I set the autocorrelation parameter to 0.45[61]. The share of endowment that goes to the borrowers (ν) is chosen to match the net fixed-income position for each household in the Survey of Consumer Finance (SCF). I take the values estimated in (Elenev et al., 2016)[62] Based on their classification borrowers receive 38% of aggregate income. I set $\nu = 0.40$.

Exogenous policy rate First introduced in the ARM mortgage rate equation, (3), the exogenous $i_t(\mathcal{Z}_t)$ is the third source of aggregate risk in the economy. I assume that $i_t(\mathcal{Z}_t)$ is perfectly correlated with the aggregate endowment shock. This assumption is based on the fact that short-term policy real rates[63] are on average 0.9%, if I use data from 1986 until 2020[64], while during recessions this policy rate equals 0.04%, and during expansions it equals 1.58%. I set the autocorrelation parameter of this shock to equalize that of the aggregate income process, 045. Then I pick the following values for the short-term policy real rates in the model to closely match those in the data: 0.01%, 1.1%, and 1.6%.

Government In the benchmark economy, I set the value ϕ^{fee} to match the fraction of mortgages that are securitized by the GSEs and Ginnie Mae (Agency-MBS). A value of $\phi^{fee} = 63$ bps[65] produces an average of insured FRMs of 71.5%. Currently, this numbers is around 70% in the data, see mortgage funding for the US in Figure 10. Finally, I assume an exogenous government expenditure (G^{gov}) in the budget constraint of the government, equation (50). I set the value of $G^{gov} = 0.15$, to match the government expenditure to GDP ratio of 15.9% for the US (I normalize the aggregate income to 1).

[60]This in turn set the average loan-to-value ratio for non-refinancers at around 61%.

[61]I Normalize the middle value of the aggregate endowment to Y=1.

[62]They define net fixed income as total bond and bond-equivalent holdings minus total debt. If this position is positive, they consider a household to be a saver, otherwise it is a borrower.

[63]This paper is a real model, I do no model inflation.

[64]I define the short-term policy real rates as the difference between the nominal Federal Funds Rate and US Core Inflation.

[65]This number closely resembles the guarantee fee charged on GSEs MBS of 58 bps, see Appendix 7.3.

4.2 Solution Method

I solve the model numerically using a global projection method. Apart from the exogenous state variables, my model features various endogenous aggregate state variables, which include the wealth distribution across the different optimizing agents, mortgage debt outstanding (needed to keep track of the long-term mortgages), and mortgage payments (required to solve carefully a model with a contract choice coming from long-term mortgages)[66]. The model's equilibrium can be characterized using two types of functions: transition functions over these endogenous state variables which map today's state into probability distributions of tomorrow's state, and policy functions that determine agents' decisions and prices given the current economy's state. This is similar to the solution method proposed by (Elenev et al., 2021).

5 Results

In this section I present the results from the quantitative model. I will show three sets of results. First, I will compare the stationary distributions from various economies to show the effect of the credit guarantees and the endogenous mortgage choice on financial stability and welfare. Second, I will compare the behavior of these different economies under a housing recession shock that features both low endowment realizations and high house price shock dispersion. Third, I will construct the menus of mortgage rates priced by the intermediary in these economies.

5.1 Main Results: Mortgage Choice and Credit Guarantees

To quantify the effect of eliminating the credit guarantee, I use my model to compute three different economies. In the first economy I match the guarantee-fee observed in the data and calibrate the model to the US economy, the *baseline economy*. Around 70% of FRMs in the economy come with a credit guarantee, similar to the data. FRMs make up the majority of outstanding mortgage debt, and borrowers optimally choose an FRM share of 71.5%[67]. For the first counterfactual economy I increase the guarantee-fee such that there are no insured FRMs, and allow the borrower to optimally choose the ARM share, the *optimal ARM share economy*. This economy features a 56.7% ARM share. Finally, the second counterfactual economy, the *constrained ARM share economy*, is one in which I do not allow for mortgage insurance (as in the

[66]In the appendix 7.26 I show the transition equations for all the state variables in the economy

[67]Around 90% of mortgages in the US come with a long term fixed-rate for the entire term. More than 75% are 30 year fixed-rate mortgages, while the rest are mainly 15-year fixed-rates mortgages. In this paper, I calibrate the share to the 30 year fixed-rate mortgages share.

optimal ARM share economy) and also constrain the FRM share to not go below 71.5% (as in the *baseline economy*)[68]. The rationale behind this economy is to eliminate the credit guarantee while not allowing the mortgage choice to be endogenous. Tables 2 - 4 present the results from the three different economies. These are the solutions from the stationary distribution for each respective economy[69].

Adjustable rate mortgage share The ARM share for new originations[70] in the *baseline economy* is 28.5%, while for the *optimal ARM share economy* is 56.7%. When I condition on the economy facing a housing recession[71], these fractions become 26.7% and 56.4%, respectively. For the *optimal ARM share economy* the intermediary barely modifies its mortgage mixture during recessions, while for the *baseline economy*, around 2% fewer ARMs are originated. Housing recessions are persistent which makes the bank optimally increase its share of the 'safe' mortgage temporarily. Furthermore, the standard deviation for the ARM share is larger when the credit guarantee is active, rationalized by the behaviour during bad times.

I also compute the ARM share for mortgages outstanding (excluding new origination)[72]. For the *baseline economy* the share equals 29.3%, while for the *optimal ARM share economy* it equals 56.3%. Relative to new originations, the share is around 1% higher for the *baseline economy.* As I show later, this is a consequence of the 'large' mortgage default coming from FRMs, which implies that more FRMs will be depleted every period. For the *optimal ARM share economy* this reverts, but barely changes 0.1%. Since mortgage leverage in this economy now mainly comes from ARMs, a larger ARM default rate is expected since leverage affects mortgage default directly.

By construction, for the *constrained ARM share economy*, the ARM share for new originations will *exactly* equal 28.5%. The standard deviation is zero because the intermediary always hits the constraint since optimally it would choose the mortgage shares in the *optimal ARM share economy.*

Borrowers The average borrower loan-to-value (LTV) and payment-to-income (PTI) ratios increase with accessibility to the credit guarantee. This is a consequence of the low mortgage rates and the elevated house prices in the *baseline economy.* The LTV for new originations in this economy is around 87%[73], while the

[68]The only change I do in the model is add a constraint to the Financial Intermediary problem such that the share of FRMs being originated at any point in the state space is no lower than 71.5%.

[69]The tables report unconditional means and standard deviations of the main outcome variables from a 20,000 period simulation for each different economy.

[70]In the model, this refers to the share of ARMs for refinancers.

[71]This features both low endowment realizations (Y_t) and high house price shock dispersion (σ_ϵ^{high})

[72]In the model, this refers to the share of ARMs for non-refinancers.

[73]The median LTV at origination is 80% for the conforming loans securitized by the GSEs, while for the Ginnie Mae loans it is around 97%. Taking the shares for each of these agencies in the US mortgage market, as observed in Figure 10, the median

LTV for mortgages outstanding (which excludes new originations) is 60.6%. This results in a total LTV (including all mortgages) of 67.1%, which closely resembles the data[74]. Similarly, I show the results for PTI ratios. The PTI for new originations in the *baseline economy* is around 36%[75]. This in turn results in a PTI ratio for old mortgage debt of 20.7%[76]. The total PTI ratio is then 23.4%.

Once I eliminate the credit guarantee on FRMs, the size of the mortgage market shrinks; see Table 3. The mortgage market also becomes safer, the LTV ratio at origination drops by 5 percentage points, while the PTI ratio at origination drops by 3 percentage points in the *optimal ARM share economy*. This spills over to the total LTV and PTI ratios of the economy, the total LTV ratio drops 3%, while the PTI ratio drops around 1.5%. Why do the LTV ratios at origination drop more? There is more default in the insured economy, which depletes a larger fraction of the debt, on average, each period (see the LTV and PTI ratios for old debt outstanding). Furthermore, the standard deviation for both the LTV and PTI ratios drops as the insured fraction of FRMs increases. This reflects the fact that leverage is less reactive to economic fluctuations in the *baseline economy*. With a expensive credit guarantee, banks might need to deleverage more as a response to a crisis, while in the cheaper credit guarantee case the financial intermediaries are not require to deleverage so much since they will just off-load the credit risk to the GSEs.

It is difficult to understand the effect of mortgage choice by comparing only those two economies, since deleveraging could be driving the entirety of the results. To disentangle the effect of the endogenous mortgage choice, I turn to the *constrained ARM share economy*. By not allowing the ARM share to adjust, the LTV and PTI ratios would drop even more (the size of the mortgage market would also be reduce more, see Table 3). The LTV ratio at origination drops by 6.5% points, while the PTI ratio at origination drops by 3.7%. However, as we will see later, this does not mean the economy becomes safer relative to the *optimal ARM share economy*. This underscores the significant impact of mortgage choice; borrowers recognize the pricing benefits of ARMs and are induced to leverage up when allowed to choose their optimal mortgage mixture[77]. Finally, the standard deviation for both the LTV and PTI ratios increase in the *constrained ARM share economy* relative to the *optimal ARM share economy*. This shows that leverage is more volatile under an

LTV at origination that comes with some form of public guarantee is around 86%.

[74]LTV on the SCF is around 64%, see Diamond and Landvoigt (2021).

[75]The median PTI at origination is 36.3% for the conforming loans securitized by the GSEs, while for the Ginnie Mae loans it is around 41%. Taking the shares for each of these agencies in the US mortgage market, as observed in Figure 10, the median LTV at origination that comes with some form of public guarantee is around 38%.

[76]This is an 'aggregate' statement. For example, individually mortgagors will keep paying a fixed payment if they hold a FRM for the entire term of the contract. However, in the aggregate the Intermediary will receive a smaller payment on its mortgages due to default, prepayment, and normal amortization.

[77]A similar result was found in (Guren et al., 2018).

economy with a majority of FRMs, for instance during downturns higher deleveraging will be required.

Mortgage Default Recall that the mortgage default for the borrower is induced by 'market priced' LTV ratios, which also include mortgage payments. Equations (93) and (96) show the default threshold expressions for ARMs and FRMs, respectively. The results from the previous section would therefore imply that we would see larger default rates under the *baseline economy*. I show here that this is the case. Table 2 shows the overall mortgage default rate, while Table 4 show the default rates by mortgage type. The overall mortgage default rate for the baseline economy is 1.8%, which is composed of a 4.3% default rate from new originations which are highly leveraged and a 1.3% mortgage default rate for the old mortgage debt. I also show the default rates conditioning on housing recessions, since this will become relevant for the comparison across mortgage types. The overall default rate for those periods is 3.9%, composed of a 8.5% default rate from new originations, and a 2.9% default rate for the old mortgage debt. Table 4 shows the default rates by mortgage type. Due to the high leverage of the economy, and the fact that most of it comes from FRMs, default rates are larger than those for ARMs[78]. The overall default rate for FRMs is 1.8%, while that for ARMs is 0.7%. During housing crisis, the default rate for FRMs is 3.9% and that for ARMs is 1.8%.

As before, these numbers include the leverage effect and the mortgage choice effect. To disentangle these effects I study the other two economies without credit guarantees. In the *optimal ARM share economy*, the default rate is 0.8%, 1% less relative to the *baseline economy*. This difference is starker during housing recessions since the default rate drops to 1.9%, which is 2% less than the *baseline economy*. New originations suffer the biggest drops since in this economy newly originated mortgages come at lower LTVs and at a higher mortgage rate (as I will discuss later). Unconditionally, the default rate goes from 4.3% to 1.0%; in the housing crisis, the default rate drops from 8.5% to 2.4%. Next, I study the default rate from the *constrained ARM share economy*. The key takeaway is that even with lower LTV and PTI ratios, relative to the *optimal ARM share economy*, I observe larger default rates in the *constrained ARM share economy*. These defaults are driven by the endogenous mortgage choice. Unconditionally, the default rate is 0.9%, 0.1% higher relative to the *optimal ARM share economy*. During housing recessions the default rate is 2.1%, which is 0.2% more that in the *optimal ARM share economy*.

In Table 4 I show the default rates for each mortgage type. ARMs show both less default overall and particularly less default during housing recessions. Lastly in Figure 2, I also show that mortgage default

[78] Recall that in the model, there are two 'kinds' of default. Either defaulting on one mortgage type uniquely or defaulting on both mortgages. This implies that the overall default rate will match the default rate of the mortgage type that has the largest default rate.

rates are less volatile across aggregate states of the world in the *optimal ARM share economy*. The biggest drop in volatility comes from switching from the *baseline economy* to the *optimal ARM share economy*. Nevertheless, in the *constrained ARM share economy* volatility is larger, too. This is due to the larger default rates that FRMs experience during downturns.

Mortgage Rates and Prices One of the objectives of providing a public credit guarantee is for the government subsidy to lower mortgage interest rates, thereby providing affordability. The model rationalize this, too. In the *baseline economy*, the mortgage rates on FRMs and ARMs are on average 3.6% and 3.7%, respectively. The mortgage rates are lower for FRMs even with a high FRM leverage and the large mortgage default rates studied before. Therefore the fact that the rate for ARMs is higher than that for FRMs reflects the pricing of the credit guarantee. When I eliminate the credit guarantee on FRMs, both mortgage rates increase, which implies that the benefit of holding insured FRMs spills over to the ARM rate. In particular, for the *optimal ARM share economy* the mortgage rate on FRMs increases to 4.4%, an increase of 80 bps relative to the *baseline economy*; while that for ARMs goes to 4.0%, an increase of 30 bps. FRMs are now more expensive. Furthermore, the volatility of the ARM rate remains more or less the same, while the volatility for the FRM rate increases. This suggest that the credit guarantee not only lowers mortgage rates but also makes them less sensitive to economic conditions.

When I do not allow for the mortgage choice to be endogenous, the mortgage rate on FRMs increase even further to 4.6%, while that for ARMs drops to 3.6%. Therefore, going from the *constrained ARM share economy* to the *optimal ARM share economy* reflects the relevance of the endogenous mortgage choice. That is, if I did not allow for the ARM share to be re-optimized the majority of mortgages would come at a fixed rate and on average the mortgage rates would be 100 bps larger than in the insured economy, while allowing for the endogenous mortgage choice means having a economy with a majority of ARMs, which are only 40 bps larger than the *baseline economy*. These numbers demonstrate that eliminating the credit guarantee while allowing for borrowers to select into different mortgage products would not only keep the mortgage provision safe, but also prevent a substantial increase in mortgage prices.

Furthermore, the risk-free interest rate is higher in the *baseline economy*. In this economy, a large supply of deposits from the intermediary is required to fund the large share of mortgages in the economy (see Table 3), this large supply brings down the price of deposits which increases the risk-free interest rate on deposits. On the demand side, we have the savers (who are the marginal buyers in this market) demanding precautionary savings. Given the low risk aversion assumed for these agents ($\gamma = 1$) I conclude that in this model the supply side forces overcome the demand side ones, thus increasing the interest rate under an economy with a credit guarantee and a highly leveraged intermediary[79].

Finally, the model also makes predictions on house prices. In the *baseline economy*, mortgagors are faced with low mortgage rates which increase the demand for housing. Given the fixed housing supply, increased housing demand results in higher house prices compared to the economies without the credit guarantee, around 1% higher relative to the *optimal ARM share economy*. Nevertheless, the house price drops even more for the *constrained ARM share economy* due to the surge in mortgage rates, such that house prices are 1.5% higher in the *baseline economy*. This is suggestive evidence that house prices would endogenously react to mortgage choice through mortgage rates which vary substantially with the availability of mortgage products. Restricting the choice set implies larger drops.

Financial Intermediaries Financial intermediates make long-term mortgage loans to impatient borrower households and borrow short-term from patient depositor households. Table 3 documents some relevant statistics on the financial intermediary. In the *baseline economy*, the credit guarantee provided by the government allows intermediaries to shield their balance sheet from the credit risk of FRMs. The consequences of this is are (i) low bank wealth (or equity), (iii) a highly leveraged balance sheet, (iii) the capital constraint on average is not binding due to the benefits of the credit guarantee, but becomes tighter (relative to the other economies) during downturns, and (iv) a larger balance sheet composed of mortgages.

Relative to the *optimal ARM share economy*, the bank's equity is around 10% lower when the insurance is available, and the bank's equity is also more volatile. During downturns this worsens since the equity is 14% lower and 50% more volatile. This is due to the fact that the mortgages originated are riskier on average in the *baseline economy*, and that only 70% of FRMs are insured, meaning that the remaining 30% does suffer the consequences of the large mortgage defaults. Additionally, the intermediary's leverage is 3% larger compared to that which is observed in the *optimal ARM share economy*; the intermediary does this in order to increase their return by taking advantage of the reduced default risk. Moreover, I measure the tightness of the capital constraint by presenting the average value of the multiplier of this occasionally binding constraint. The average value in the insured economy is 0.02, while in the *optimal ARM share economy* it is larger and equal to 0.067. The financial intermediary's collateral constraint becomes tighter when the public subsidy becomes unavailable in part because of the higher default risk suffered and in part because of the higher regulatory requirements on ARMs (and non-insured FRMs) relative to the insured

[79] In (Elenev et al., 2016), the authors get the opposite results by assuming highly risk averse savers.

FRMs.

When I do not allow for the mortgage choice to be endogenous, the equity is lower compared to the *optimal ARM share economy*. In particular, in the *constrained ARM share economy* equity is 1.7% lower on average, and 2.7% lower during housing recessions. This is mainly due to the higher default rates (especially during downturns) coming from FRMs. This is true even when the intermediary's leverage is 1% lower in the *constrained ARM share economy*. Finally, the capital constraint becomes slightly more binding in this economy, again reflecting the larger default rates.

Secondary Market Prices The values of the secondary market prices for the financial intermediary are a key component of mortgage pricing (see equations 36 and 37 in the main text). These are required to price the future risks faced by the intermediary at the moment of origination, i.e. a consequence of modelling long-term contracts. Table 3 shows the secondary market prices for the PO assets for both FRMs and ARMs (I do not show the prices of the IO assets, but they have similar intuition). Relative to the *optimal ARM share economy*, the PO secondary market price for FRMs is around 1% larger on average and around 2% larger during housing recessions in the *baseline economy*. This helps bring down the mortgage rates on FRMs at origination, and the price of the future credit risk is not internalized due to the credit guarantee. Furthermore, the credit guarantee aids in reducing the volatility of these prices. For the PO secondary market price for ARMs, the story is different. Relative to the *optimal ARM share economy*, these prices do not drop as much, and the volatility actually increases.

I can decouple the leverage effect and the mortgages choice effects. Therefore, I compute these prices for the *constrained ARM share economy*. The PO secondary market price for FRMs drops even further (compared to the *optimal ARM share economy*). In particular, and relative to the baseline economy, the decline during normal times is 2.1%, and 2.9% during housing recessions. This clearly impacts mortgage rates substantially (100 bps in the FRM rate). Since the ARM leverage is low in this economy, these prices do not vary substantially, but they do increase relative to the *optimal ARM share economy*. These prices try to reflect the liquidity that the government intends to provide to the mortgage with the agency-MBS.

Consumption and Welfare Table 4 shows the consumption and welfare consequences on borrowers and savers of removing the credit guarantee and endogenizing the mortgage choice. Overall, borrowers' consumption and welfare increase in the *optimal ARM share economy*, relative to the *baseline economy* with credit guarantees. The biggest change, however, comes from the decrease in consumption and welfare volatility. The economy with credit guarantees requires borrowers to deleverage highly during downturns which is costly in consumption terms due to the long-term mortgage assumption. Furthermore, the default rates are larger in the *baseline economy*, and by assumption this is also costly in consumption terms. The model, therefore, predicts that the benefits of lower mortgage rates are smaller than the costs of the balance sheet instability. Notice however that consumption and welfare also increase in the *constrained ARM share economy*, relative to the *baseline economy*. This indicates that the big welfare benefits come from households holding safer mortgage debt in general, which makes their balance sheet safer. The consumption and welfare volatility also decreases in this economy.

Nevertheless, comparing the *optimal ARM share economy* with the *constrained ARM share economy* shows that there is some extra benefit from allowing an endogenous mortgage choice. In particular, the pro-cyclicality of ARM payments hedges households against big drops in consumption during downturns. This extra benefit is non-trivial quantitatively.

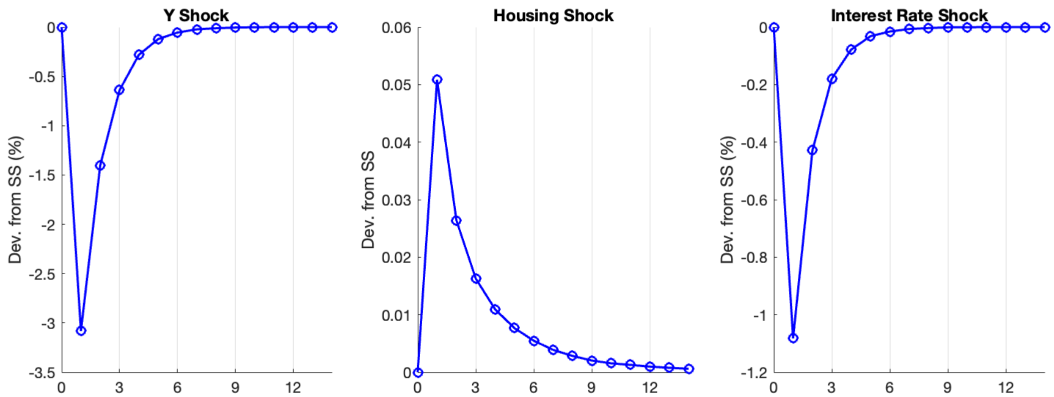
Savers do suffer from removing the credit guarantees. Their consumption and welfare drops, though it is worth mentioning that these drops are relatively small compared to the borrower. The decrease in consumption and welfare can be explained by the drop in (i) the risk-free rate since savers are the main holders of deposits. Furthermore, (ii) housing prices also decrease and savers also hold a large fraction of the housing in the model. (iii) They do not benefit (directly) from holding safer mortgages. However, the volatility in consumption and welfare of savers do decrease by removing the credit guarantees, similar to borrowers.

5.2 Results: Housing Recession

This section present the second set of quantitative results. I show the behavior of some model variables under a *housing recession* that features both low endowment realization and high house price shock dispersion. The goal is to study the behaviour of the economy under aggregate shocks. In particular I will show impulse response functions for each of the three economies studied in the last section. Figure 2 shows the value of the shocks that I will be using to study the housing recession. These will be the same shocks faced by all three version of the economy. Aggregate income will drop by 3%, the exogenous policy rate will follow and drop around 1%, and the volatiliy of the housing risk shock will go from $\sigma_\epsilon^{low} \rightarrow \sigma_\epsilon^{high}$[80].

[80] All these shocks are model as Markov chains with some persistence, as explained in the calibration section 4.

Figure 2: IRFs (Aggregate shocks)



The IRF plots are computed by simulating the economy 5,000 times for 15 periods, the average values per period are plotted. The simulations are initialized at the ergodic distribution of the endogenous states, the mean income level, and in the low-housing risk state. *Dev. from SS* refers to raw difference with respect to the ergodic distribution values. *Dev. from SS (%)* refers to percentage difference with respect to the ergodic distribution values.

5.2.1 Response to Aggregate Shocks: Mortgage Market Variables

Figure 3 shows the impulse response functions for bank equity, mortgage rates, mortgage default rates, and mortgage originations. The color coding references each of the different economies. The blue circled lines refer to the *baseline economy*, the black dashed lines refer to the *optimal ARM share economy*, and the red crossed lines refer to the *constrained ARM share economy.* Bank equity drops around 30% in the *baseline economy*, while the drop is larger for the other two economies, around 36%. This is consistent with the credit guarantees providing some insurance against credit risk. Notice however, that the speed of convergence is much more slower for this highly leveraged economy. As we will in the remaining plots this will be a reflection of mortgages rates remaining low which in turn incentivizes leverage to remain at high levels even after the housing crisis occurs. Since the housing risk shock is persistent this will force default rates to remain high for some time, this explains the sluggish convergence of the bank's equity. For the *optimal ARM share economy* and *constrained ARM share economy* bank equity recovers faster, this is a consequence of precautionary deleveraging in crises. Bank equity drops slightly less for the *optimal ARM share economy*, 1% less.

Next, I study how mortgage rates and mortgage default react to the housing recession experiment. In the *baseline economy* mortgage rates remain (relatively) low. FRMs increase by 86 bps, while ARMs increase by 95 bps. The credit guarantee benefits FRMs more, but since the entire balance sheet is shielded ARMs also benefit indirectly from the credit guarantee[81]. Notice however that default rates are 50% larger in the

[81] See section 5.3 where I compute the entire menus for both mortgages.

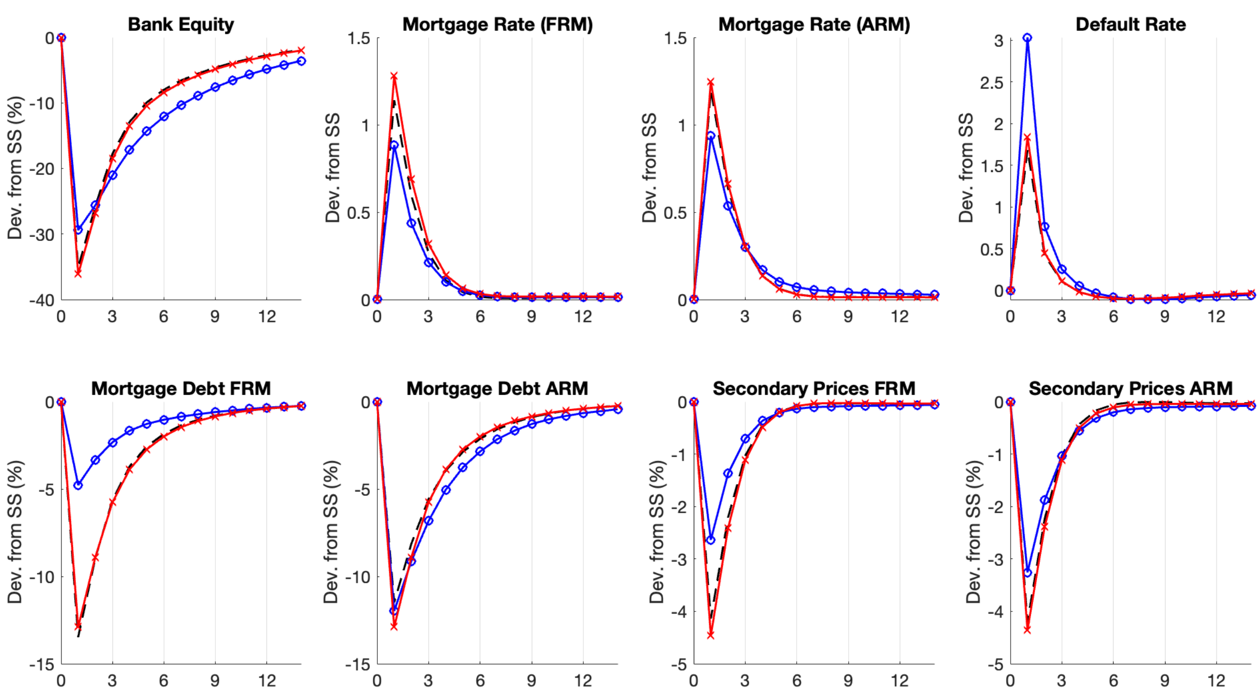
baseline economy, relative to the other two economies. Moreover, default rates remain at high levels for longer periods. The only way such high (average) default can co-exist with low mortgage rates is through the credit guarantee. Comparing the *optimal ARM share economy* with *constrained ARM share economy* we also see some interesting differences.

At the moment of impact, average default rates increase by 1.68% in the *optimal ARM share economy*, while the increase is 1.84% for the *constrained ARM share economy.* Since there is no credit guarantee in these economies, credit risk will be priced correctly. In particular, mortgage rates increase more in the economy where I do not allow for an optimal ARM share. The rate on FRMs increases by 130 bps, while that for ARMs increases by 122 bps. This difference can be explained by the larger share of FRMs in this economy. Contrary to these effects, in the *optimal ARM share economy*, the mortgage rates on FRMs increase by 113 bps, and for ARMs, the increase is 118 bps. Again, this difference can be explained by the larger share of ARMs in this economy. Therefore, I can conclude that mortgage rates will increase more in the *constrained ARM share economy*, not only because of the bigger increase in mortgage rates but also because there are more FRMs in that economy, and that is this is precisely the mortgage rate that increases the most. Adjustable rate mortgages provide some insurance against the housing crisis given the procyclicality of its mortgage payments.

The next panels show the percentage change in the value of mortgage face values at origination (i.e., the size of the mortgages being issued). There is a big difference between FRMs and ARMs in the *baseline economy.* The intermediary decides to deleverage more on its ARM holdings, the size of ARMs being originated drops 12% while that for FRMs decreases only by 4.8%, taking advantage of the credit guarantee. The deleveraging is greater in the uninsured economies, this makes sense from a precautionary motive perspective. Nevertheless. comparing across these two economies there are some differences. In the *optimal ARM share economy*, the intermediary also strategically chooses how much to deleverage from each mortgage. The size of ARMs drop by 11.4% while that for FRMs drop by 13.5%. Holding more ARMs is a better idea during downturns. In the *constrained ARM share economy*, the drop is 13% for both mortgages. Since there are more FRMs in this economy the size of mortgages (in the aggregate) drops by a larger amount in this economy.

Finally I present the impulse response function for the secondary market prices (these are the PO prices). These prices rationalize (partially) the mortgage rates behavior studied before. For the *baseline economy*, the secondary market prices for FRMs drop by 2.6%, while for ARMs they drop 3.2%. These prices react to future expected default. For FRMs the small (relative) drop can be explained by the credit guarantee, for ARMs the small (relative) drop can be explained by ARMs not being the main mortgages in this economy coupled with the size of ARMs shrinking for some time which of course impacts future default. Therefore the credit guarantee provides insurance to the FRM market, but also allows ARMs to be temporarily substituted and therefore reduce the risk of the balance sheet even further. Lastly, the drop for the secondary market prices in the *constrained ARM share economy* is larger, 4.5% for FRMs and 4.4% for ARMs. This compares with the price reductions of 4.1% for FRMs and 4.2% for ARMs in the *optimal ARM share economy*. Mortgage choice plays a role here, the intermediary optimally decides to increase its ARM holdings since they provide better hedging for the borrower and therefore lower the risk of its balance sheet. At then end, these will be reflected in the mortgage rates.

Figure 3: Impulse response functions (Mortgage Market Variables)



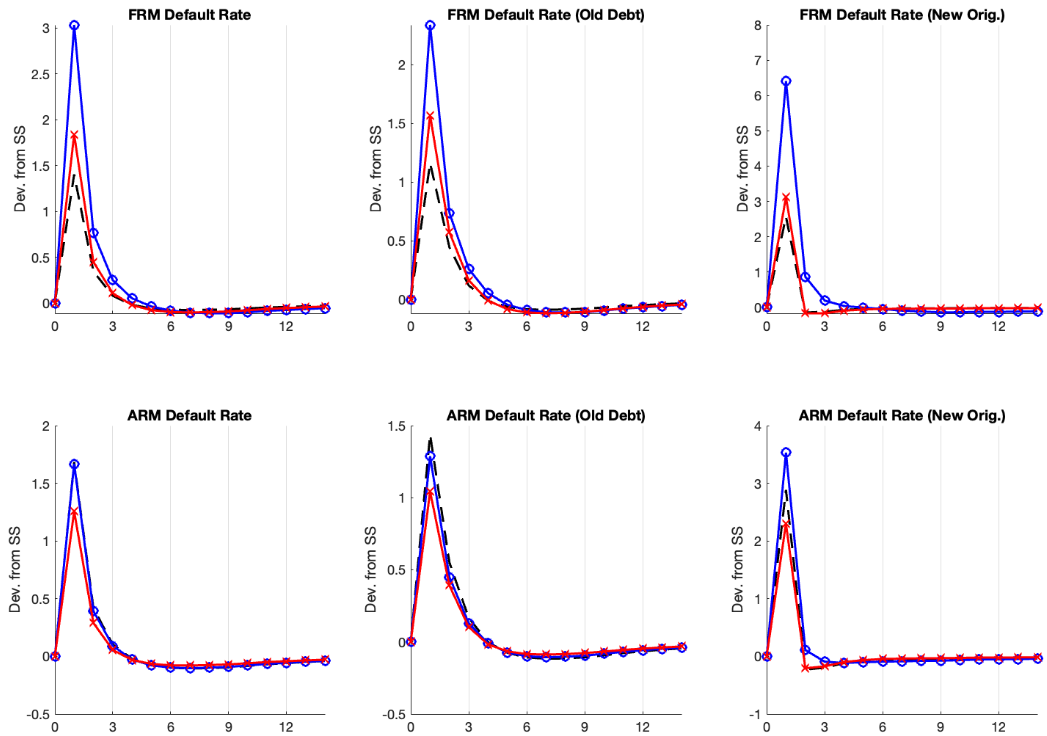
The blue circled lines refer to the *baseline economy*, the **black** dashed lines refer to the *optimal ARM share economy*, and the red crossed lines refer to the *constrained ARM share economy*. The IRF plots are computed by simulating the economy 5,000 times for 15 periods, the average values per period are plotted. The simulations are initialized at the ergodic distribution of the endogenous states, the mean income level, and in the low-housing risk state. *Dev. from SS* refers to raw difference with respect to the ergodic distribution values. *Dev. from SS (%)* refers to percentage difference with respect to the ergodic distribution values.

5.2.2 Response to Aggregate Shocks: Default Rates by Mortgage Type

Default Rates are larger in almost all cases for the *baseline economy*. The default rate for FRMs increase by 3%, while for ARMs is only 1.7%. The default that increases the most is that of new originations, since these are the highly leveraged mortgages. The default rate on newly originated FRMs is 6.4%, and for ARMs is 3.5%. Notice that the default rate on new originations drops almost immediately. This is explained by the fact that after the shocks mortgages will be smaller and safer, leverage decreases. Therefore *new originations* will hold very small risk. Nevertheless, since mortgages are long-term contracts the history of highly leveraged mortgages will be be reflected in the default rate for old debt. The default rate for FRMs goes up to 2.4% and to 1.3% for ARMs. In this economy, the FRM share is larger, the impulse response function for old debt FRMs is more persistent than its counterpart for the other economies.

We now turn our attention to the other two economies. For the *optimal ARM share economy*, the default rate on FRMs is 1.4% and for ARMs is 1.7%. For the *constrained ARM share economy* the default rate increases by 1.9% for FRMs and by 1.3% for ARMs. The overall default rate then is larger in the *constrained ARM share economy* since the intermediary is forced to hold a larger share of FRMs that default at a higher rate. If given the option, the intermediary would hold more ARMs to alleviate these constraints. This logic holds if I study new originations and debt that comes from previous originations. Finally, it is worth mentioning that for the uninsured economies the deleveraging (as we will see in the next section) is so large that the default rates for new originations go back to its initial stationary distribution value in the second period. That is, the intermediary measures the riskiness of the economy and deleverages enough to go back to its initial default rates. This is not true for the *baseline economy* which shows a slightly slower process.

Figure 4: Impulse response functions (Default Rates by Mortgage Type)



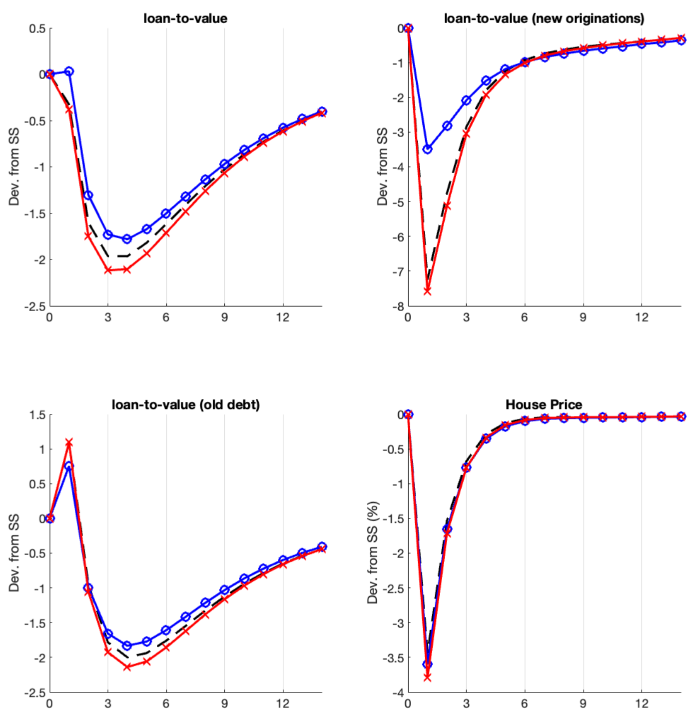
The blue circled lines refer to the *baseline economy*, the **black** dashed lines refer to the *optimal ARM share economy*, and the red crossed lines refer to the *constrained ARM share economy*. The IRF plots are computed by simulating the economy 5,000 times for 15 periods, the average values per period are plotted. The simulations are initialized at the ergodic distribution of the endogenous states, the mean income level, and in the low-housing risk state. *Dev. from SS* refers to raw difference with respect to the ergodic distribution values. *Dev. from SS (%)* refers to percentage difference with respect to the ergodic distribution values.

5.2.3 Response to Aggregate Shocks: Mortgage Leverage

Finally, I present the impulse response functions for the behaviour of mortgage leverage in Figure 5. Under the *baseline economy*, the LTV for new originations only drops by 3.5%, and slowly reverts back to the stationary distribution. The credit guarantee allows the intermediary to keep originating risky mortgages even during downturns. For the *optimal ARM share economy* the drop is of 7.2%, for the *constrained ARM share economy* the drop is 7.6%. Therefore, without mortgage choice, we would observe significant deleveraging during crisis periods. All these declines are larger compared to the reduction in house prices, shown also in Figure 5. For debt that was originated in the past, the LTV increases at the moment of impact, this is a unique feature of models with long-term mortgages. It is worth noting that the largest increase in this LTV comes from the *constrained ARM share economy*. Mortgage rates are the highest in this economy, this reduces the demand in the mortgage market which lower house prices (the housing supply is fixed). Therefore, even for initial lower levels of debt, leverage of past debt increases the most in this economy, 1.1%. In the *baseline economy*, mortgage rates are low and therefore the demand for housing does not drop so abruptly. In the *optimal ARM share economy*, the endogenous mortgage choice allows the intermediary to keep mortgage rates relatively low which allows the demand for housing to not drop as much, the increase in the LTV of old debt is 1%.

Equally important are the long-term effects of mortgage choice. If the intermediary is allowed to choose its optimal mixture of mortgage contracts, the loan-to-value ratio of the economy does not drop as much as in the *constrained ARM share economy.* The overall LTV of the economy never goes below 2% of its initial value in the *optimal ARM share economy*, while for the *constrained ARM share economy*, this value stays below 2% for a couple of periods and converges more slowly.

Figure 5: Impulse response functions (mortgage leverage)



The blue circled lines refer to the *baseline economy*, the **black** dashed lines refer to the *optimal ARM share economy*, and the red crossed lines refer to the *constrained ARM share economy.* The IRF plots are computed by simulating the economy 5,000 times for 15 periods, the average values per period are plotted. The simulations are initialized at the ergodic distribution of the endogenous states, the mean income level, and in the low-housing risk state. *Dev. from SS* refers to raw difference with respect to the ergodic distribution values. *Dev. from SS (%)* refers to percentage difference with respect to the ergodic distribution values.

5.3 Mortgage Rate Menus

In this section I present the third quantitative experiment of the model. The goal is to compute the mortgage rate menus for both mortgage types under the *baseline economy* and the *optimal ARM share economy*. A menu defines the mortgage contracts available to a borrower who then selects an optimal contract among the entire menu. Therefore, the goal of this section is to compare the mortgage rates offered by the financial intermediary as a function of:

1. some borrower's characteristic, in particular I will plot each of the menus as a function of the loan-to-value ratio at origination;
2. the fraction of the FRMs in the balance sheet that are sheltered from credit risk;
3. the mortgage type, either FRM or ARM.

To compute the menu I start by calculating the mean value LTV (μ_{LTV}) using the ergodic distribution. The following procedure focuses on the rate offered to a (counterfactual) borrower (b) with a $LTV^b \in [\mu_{LTV}^{min}, \mu_{LTV}^{max}]$[82]. The procedure is as follows:

1. (Outer loop) For each point in the state space s (i.e. particular combination of state variables)
   (a) Use prices and aggregate quantities that solve the system for equations in that particular point in the state space s.
   (b) (Inner loop) For each LTV^b (i.e. a counterfactual borrower) solve the non-linear equation that pins down the mortgage interest rate using the FOC for newly originated mortgages. For the FRMs, this is described in (158); for the ARMs, this is described in (159).
2. Compute an average interest rate for each counterfactual borrower using as weights the probability of each point in the state space from the ergodic distribution. Each (weighted) interest rate represents a point in the menu. Do the same procedure for the FRM contract and for the ARM contract.

Figure 6 shows menus for ARMs and FRMs under two values for ϕ^{fee}. First, the left panels plot the menus when $\phi^{fee} = 63$ bps and therefore 70% of the FRMs in the balance sheet come with a credit guarantee. The right panels plot the menus when $\phi^{fee} = 500$ bps, and therefore the balance sheet of the bank is entirely

[82] For all menu contracts, I pick the following values for the interval limits: $\mu_{LTV}^{min} = 0.75$ and $\mu_{LTV}^{max} = \mu_{LTV} + 0.11$. For the credit surface with $s = 0\%$, the LTV $\in [0.75, 0.928]$ since $\mu_{LTV} = 0.818$. For the credit surface with $s = 70\%$, the LTV $\in [0.75, 0.998]$ since $\mu_{LTV} = 0.888$.

unprotected against credit risk in this case. For both cases I show the spread between the FRM rate and the ARM rate in the lower panel. The small triangles in each line represents the solution from the stationary distribution, which was described in section 5.1.

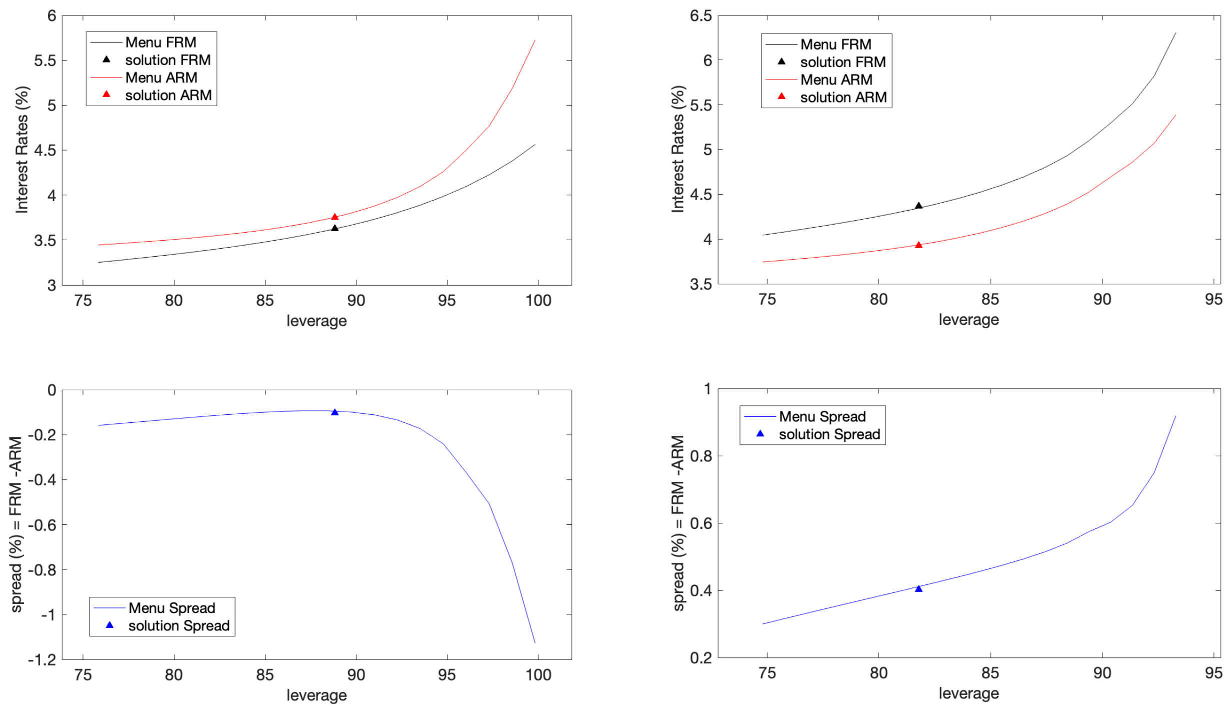
What stands out from the menus for the case when the guarantee is active is the flatness of the menu contract for FRMs. At an LTV of 75%, the FRM offered is 3.25%, while at an LTV of 100%, the FRM offered is 4.55%. Only 130 bps between these two very different risk profiles. This looks very different for the ARMs, the mortgage rate for ARMs at an LTV of 75% is 3.44%, while the ARM rate at an LTV of 100% is 5.72%. A difference of 228 bps. This results in a spread between FRMs and ARMs of around -15 bps at an LTV of 75%, while the spread is -116bps for an LTV of 100%. This is suggestive evidence that by absorbing the credit risk from the financial intermediary, the GSEs allow lenders to charge similar mortgage rates regardless of the credit risk coming from heterogeneous mortgagors if they originate a FRM. Therefore, the government credit guarantee subsidizes the riskier borrowers disproportionately since this is where the credit risk is greater.

What happens to these menus when the credit guarantee is eliminated? The solution is shown in the right panels. First, mortgage rates increase in general (the 'level effect'). For ARMs, the mortgage rate at an LTV of 75% is 3.74%. For FRMs, the mortgage rate at the same LTV of 75% is 4.04%. Both higher than their counterparts on the left panels. Nevertheless, in this case both mortgage contracts will 'correctly' price credit risk (the 'slope effect'). For ARMs, at an LTV of 93% the mortgage rate is 5.38%. A difference of 164 bps with respect to the 75% LTV value[83]. For the FRMs, the mortgage rate at an LTV of 93% is 6.30%. A difference of 226 bps with respect to the LTV of 75%. This contrasts the 130 bps in the model with credit guarantees. This results in a spread between FRMs and ARMs of around 30 bps at an LTV of 75%, while the spread is 90bps for an LTV of 93%. This suggests that in a world without credit guarantees the agents who would switch to an ARM are those with riskier profiles since they would benefit the most.

[83]Notice that this is also suggestive that in the model with credit guarantees the ARMs also benefit indirectly, since they show smaller credit spreads (comparing the 93% LTV and the 75% LTV)

Figure 6: Interest Rate Menus

(a) $\phi_{low}^{fee} = 63bps \implies s = 70\%$ (b) $\phi_{low}^{fee} = 500bps \implies s = 0\%$



Interest Rate Menus. x-axis: loan-to-value (LTV). y-axis: mortgage rate at origination. Plots generated by computing the mortgage interest rate at each point in the economy's state space and reporting an average over the ergodic distribution. Left panels: Model with credit guarantee active, 70% of FRMs are insured, *baseline economy*. Right panels: Model without credit guarantee active, 0% of FRMs are insured, *optimal ARM share economy*. The red lines represent menus for ARMs. The black lines represent menus for FRMs. The blue lines represent the spread between the FRM and the ARM contracts. The small triangle represent the solution from the stationary distribution, for each respective economy and mortgage type.

6 Conclusions

My model contributes to the literature on structural housing and mortgage finance models in two significant ways. First, the financial sector determines its balance sheet composition of FRMs and ARMs, providing borrowers with an endogenous mortgage choice. Second, the financial intermediary has the flexibility to choose the fraction of FRMs that will come with a credit guarantee. These two features closely resemble the US mortgage market. Moreover, these extensions prove to be crucial, illustrating how risk-sharing through public credit guarantees by the GSEs helps explain why the US is one of the few mortgage markets globally where high-LTV, high-risk borrowers can take out 30-year fixed-rate contracts.

In the model with credit guarantees, both borrowers and financial intermediaries exhibit high leverage. Mortgages are not only too large but also too risky, resulting in elevated default rates. Furthermore, the availability of inexpensive mortgages contributes to higher house prices. Borrowers and savers, however, face higher volatility in their consumption and overall welfare. Upon eliminating the credit guarantees and not allowing for mortgage choice to be endogenous (i.e., still forcing the intermediary to originate a large share of FRMs), the economy experiences reduced leverage, decreased default rates, and increased mortgage rates. This adjustment occurs because the intermediary now accurately prices the default risk of FRMs. However, holding a large portfolio of FRMs is not optimal. By allowing mortgage choice to be endogenous, borrowers in this economy predominantly choose ARMs. This choice is driven by the procyclicality of mortgage payments in these contracts, which lowers default rates, especially during downturns. In this economy, mortgage rates experience a less significant increase, mortgage credit provision does not decrease as much, the financial intermediary is better capitalized, and both borrowers and savers face less volatile consumption. Consequently, the overall economic performance during recessions is better than in a counterfactual world where the mortgage choice is not allowed to be endogenous. Policy interventions considering the elimination of the credit guarantee provided by the GSEs should carefully take into account these general equilibrium effects.

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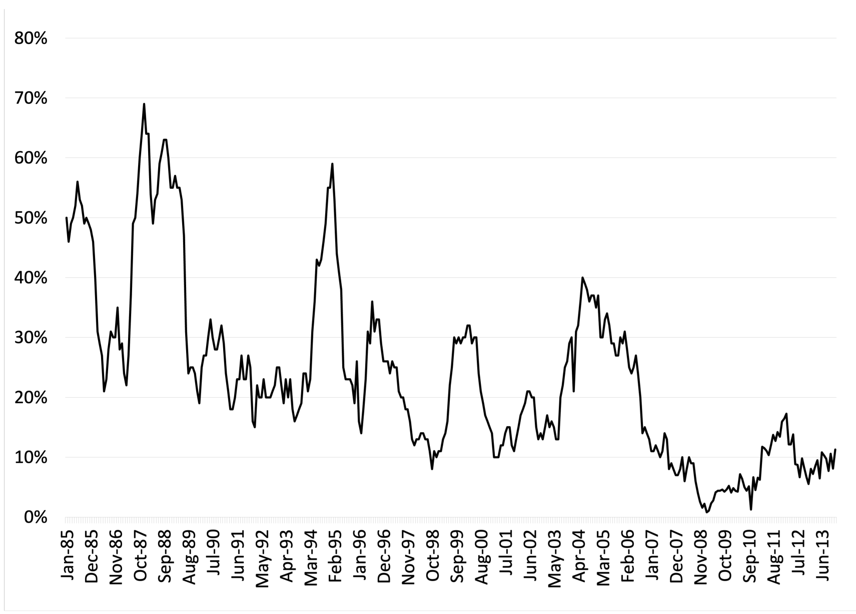
7 Appendix

7.1 Adjustable rate mortgages in the US

Figure 7 shows the share of adjustable rate mortgages for the US. The dataset used is the Monthly Interest Rate Survey from the Federal Housing Finance Agency[84]. With the survey I am able to distinguish between the different initial fixations periods. The 4 categories are: (i) from 0 - 1 year of fixation period, (ii) from 1 - 5 years of fixation periods, (iii) from 5 - 10 years of fixation periods, and (iv) over 10 years (i.e this bracked includes 15-year and 30-year fixed-rate mortgages). The definition of ARMs I use in Figure 7 is the sum of the three first three categories[85]. Figure 8 show the disaggregated data by initial fixation period[86].

Figure 7: US Adjustable-Rate Mortgage Share (% New Mortgage Originations), Monthly Interest Rate Survey, Federal Housing Finance Agency (FHFA). Discontinued in 2019.

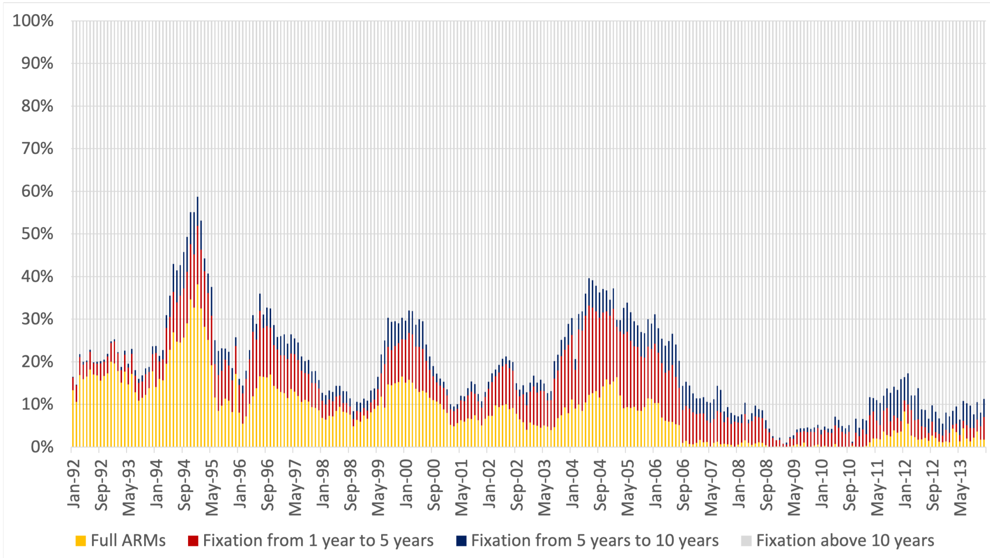


[84] Survey respondents were asked to report terms and conditions of all conventional, single-family, fully amortized purchase-money loans closed during the last five working days of the month. Survey respondents include savings associations, mortgage companies, commercial banks, and mutual savings banks.

[85] The bucket from 1-5 years are the most common ARMs used nowadays, better known as teaser-mortgages. The bucket from 5-10 years is the least popular one.

[86] The dataset for ARMs becomes very unreliable after 2013 due to the low ARM share.

Figure 8: US Mortgage by Initial Fixation Length (% Total Mortgage Originations), Monthly Interest Rate Survey, Federal Housing Finance Agency (FHFA). Discontinued in 2019.



7.1.1 Historical Context: Adjustable rate mortgages in the US

The historic ARM share for the US mortgage market exhibits great variability. Arguably, there have been three periods when ARMs have been the prevalent mortgage product in the US. First, before the 1930s when mortgages were short-term adjustable non-amortizing mortgages. Secondly, after the Saving and Loans Associations (S&Ls) crisis in the 60's and 70's. The third period is related to the housing boom of the early 2000s[87], when the composition of mortgages originated and outstanding in the United States changed drastically.

Great Depression Period Before the Great Depression, mortgage looked very different than they do today. Until the 1930s, residential mortgages in the US were available only for a short term (typically 5–10 years) and featured balloon payments of the entire principal at term. To avoid paying off the outstanding loan balance, borrowers usually tried to refinance its loan. Moreover, and relevant to this paper most loans were issued at an adjustable rate. Even though, these mortgages typically had low (relative to today) loan-to-value ratios of 50 percent or less, during the Great Depression house values in the US substantially collapsed by 50 percent. Depository institutions, who were the holders of these mortgages, refused to refinance those loans that came due which resulted in a wave of defaults, at its worst moment 10% of homes were foreclosed (Green and Wachter, 2005).

[87] The homeownership rate in the US at the first quarter of 1994 was 63.8% while at its peak in the last quarter of 2004 was 69.5%.

In response the federal government decided to intervene in the mortgage market. Three very relevant changes occurred after the Great Depression. First, in 1933 the government created the Home Owner's Loan Corporation (HOLC), this institution was created to raise funds using government-backed bonds, then it would use the funds to purchase defaulted mortgages to be later reinstated. Importantly, before putting them back in the market the HOLC modified the characteristics of these mortgages. The loans went from being short-term, adjustable-rate, balloon mortgages, to long-term (20-year) fixed-rate fully amortizing over the course of the loan's life mortgages. Around one million mortgages were replaced. Secondly, the creation of the HOLC was always assumed to be temporary since the federal government did not desire to hold mortgages in its balance sheet permanently. Consequently, the government established the Federal Housing Administration (FHA) in 1936. The goal of this institution was to create demand for the newly reinstated mortgages by providing mortgage insurance[88] to the investors. Thirdly, in 1936 The HOLC was eliminated and replaced by a newly created institution, the Federal National Mortgage Association (FNMA, later know as Fannie Mae) was created in 1938. The role of FNMA was to create a secondary market for the FHA loans. In particular, FNMA issued bonds for purchasing mortgages at par, creating a larger demand for rich investors who were unsure to put their money in loans issued in non-wealthy regions. These fast paced modifications during the 30s show that the creation of a funding instrument with the specific characteristics we see today was motivated and came tightly linked to the ability of the government to market the mortgage bonds (i.e make them more liquid and safe) that would ultimately fund the mortgage market, a first step towards the securitization wave we would observe decades later.

Saving and Loans Associations During the post-war period, the housing market grew fast alongside the newly created long-term fixed-rate fully amortizing mortgage product. Homeownership rate grew from 43% in 1940 to 55% in 1950, and finally to 63% in 1970[89]. However, the attempt of the federal government to stimulate homeownership by supporting long-term, fixed interest rate mortgages was done with funding mainly from Saving and Loans Associations (S&Ls) (Benston and Kaufman, 1997). These institutions were using short-term deposits to finance their long-term fixed-rate loans, a typical mismatch problem (Kirti, 2020; Froot et al., 1993; Rampini and Viswanathan, 2010) that is prone to disaster. These problems were exacerbated by the deposit insurance access these institutions had through the Federal Savings and Loan Insurance Corporation (FSLIC), originally intended to encourage an inflow of savings that would promote

[88] The insurance was for the full principal balance and scheduled interest payments.

[89] Historical Census of Housing Tables: Homeownership

mortgage originations[90].

Inflation spiked in the late 1960s and 1970s which altered the ability of depositories to fund these fixed-rate mortgages, inflation pushed up nominal interest rates and crippled their balance sheets. On top of that Regulation Q, a federal rule that placed a ceiling on the interest rate that S&Ls could offer customers, forced the insolvency of these institutions, given that as nominal interest rates rose, funds were withdrawn and moved towards U.S. Treasury securities.

In response the federal government decided to intervene once more in the housing and mortgage markets. Two relevant changes occurred during the 60s and 70s. First, legislation responded by removing the deposit ceilings and allowed S&Ls to invest in adjustable-rate mortgages in the early 80s. For a time in the 1980s, it appeared that the U.S. mortgage market would return to the adjustable-rate mortgages that had been common before the 1930s. Looking backward, depository institutions were worried about lending at a fixed-rate when there was a risk that nominal interest rates would rise due to the highly volatile inflation of the period. Hence, if a homebuyer chose an ARM, the depositor typically held on to it in its balance sheet, but if the homebuyer chose a FRM, then the lender would typically try to get some government insurance by selling it.

Funding through Capital Markets This takes me to the second important modification during this period. Since 1938 Fannie Mae had been the sole institution that bought mortgages from depository institution. However, in 1968, Fannie Mae was split into a private corporation and a publicly financed institution. Through the Housing and Urban Development Act, Fannie Mae (retained its name) became a private shareholder-owned corporation chartered by the U.S. Congress, its role was still to support the purchase of mortgages from savings and loan associations and other depository institutions, the main objective was to increase the trading of non-government mortgages. The publicly financed institution was named the Government National Mortgage Association (Ginnie Mae) and it still explicitly guaranteed the repayments of securities backed by FHA-foreclosed mortgages to the lender; as well as package and securitize FHA loans. Finally, to provide competition for the newly private Fannie Mae and to further increase the availability of funds to finance mortgages and home ownership, Congress then established the Federal Home Loan Mortgage Corporation (Freddie Mac) as a private corporation through the Emergency Home Finance Act of 1970.

Arguably, those are the most important regulations that the housing and mortgage markets have suffered

[90] Had it not been for federally provided deposit insurance, savers would have been less likely to put their funds into the undiversified and duration-unbalanced S&L associations. For example, see (Elenev et al., 2021) were they find a similar result.

in the last decades. The Congress's intent with the creation of Ginnie, the new Fannie and Freddie was at least in part to assure that the mortgage liquidity problems of 1966 would not repeat (Green and Wachter, 2005). These institutions would not only bring liquidity and stability to the secondary market, but also uniformity to the mortgage market. The first mortgage-backed security was issued in 1968 by Fannie Mae, and its started the shift to mortgages being funded by capital markets rather than by directly depository institution's balance sheets. Figure 10 shows how mortgages have been funded in the US since the 50s, the mortgage funding shift in the 70s can be studied there. The repercussion this had on mortgage availability can be seen in Figure 7, the FRMs originations share has been steadily increasing since the 80s. The US government has pushed for FRMs since the 40s and with a large scale securitization program, these long-term assets can be funded through capital markets as opposed to short-term liabilities, the government credit guarantee has played a key role in its development as we will see next when I describe the Great Recession period.

Great Recession Period During the housing market boom of 2001-2006, the composition of mortgages originated and outstanding in the United States changed drastically. Nontraditional mortgages with back-loaded payment structures became very popular relative to the FRMs (Corbae and Quintin, 2015; Oosthuizen and Sánchez Sánchez, 2023). While the pool of these nontraditional mortgages had a varying array of features[91], a large fraction of them were often categorized as adjustable rate mortgages. However, this is halfway true. In reality, ARMs are normally composed of an introductory period and an adjustment period. The introductory period generally lasts for about one to five years and is characterized by a low fixed interest rate, while the adjustment period is characterized by the variable mortgage payments. The low introductory rate is known as a teaser rate.

It is well-known that the housing boom was in largely driven and funded by the issuance of private-label residential mortgage-backed securities (PLS, also know as non-agency securitization). The PLS was the cornerstone of product innovation during the early 2000s, they started issuing mortgages to borrowers that the classic government-based securitization was unwilling to. Figure 10 shows that it is during this period (2000-2006) were non-agency MBS funding grows substantially. After the housing collapse, the PLS completely shuts-down. In terms of mortgage availability, this period is characterized by a resurgence of ARMs, see Figure 7. However, disaggregating the ARM share into the different initial fixation periods, we

[91]Common features observed: variable payments, balloon payments, teaser rates, interest-only periods, subprime mortgagors, no documentation, negative amortization, and lower down payments.

see an interesting pattern, see Figure 8. In the early 90s, when the economy was coming from a big wave of ARMs originations after the S&Ls collapse, the ARMs being originated are "Full ARMs" (i.e loans with very short fixation periods, less than one year), whereas during the housing boom something unusual happens and the rise in the ARM share is driven by teaser-rate mortgages (i.e. loans with medium length fixation period, 1 to 5 years)[92].

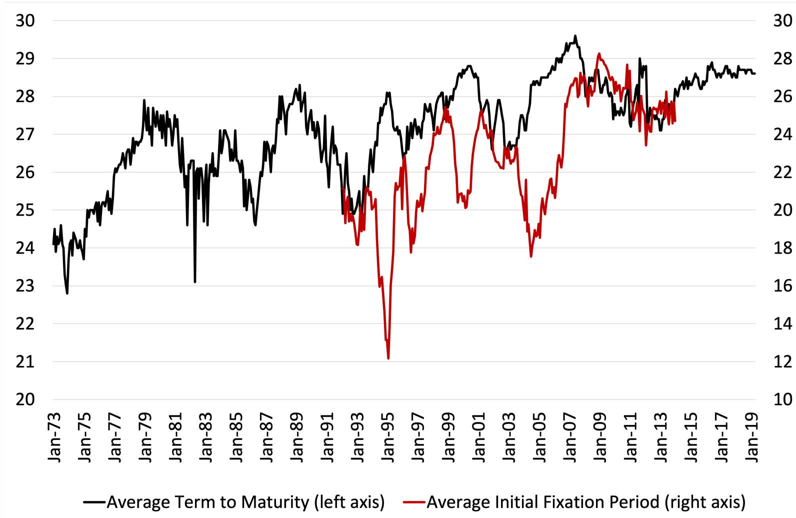
Nowadays From 2009 to early 2022, the ARM share remained very low, generally between 5 to 8 percent. Not even after the Covid crisis when inflation and interest rates have risen substantially we have seen a big resurgence in the ARMs originations, at its peak in October of 2022 it reached 13% of total mortgage applications, in July of 2023 it sits around 6.5% of total mortgage applications (Goodman et al., 2023).

7.1.2 Alternative definitions for the ARM share

Figure 9 shows an "alternative" definition for the ARM share for the residential mortgage market in the US. The left axis shows the average term to maturity for mortgages issued in the US, the right axis shoes the average initial fixation period for mortgages issued in the US. The average term to maturity has consistently increased since the early 70s from an average of 24 years to an average of almost 30 years nowadays. What is more interesting is the time series for the average initial fixation period which has also increased consistently from a value of 24 in the early 90s to around 26 in 2013, however the variance is way higher than the one for the average maturity (see axis). The big drops in the average fixation length coincides with the increases in the ARM share observed in Figure 7. The first one in the 90s, and the second one at the start of the 2000s, this last one is the effect that the teaser-rate mortgages had on the mortgage market.

[92]See Oosthuizen and Sánchez Sánchez (2023) for an in-depth analysis.

Figure 9: US Average Term to Maturity and Average Initial Fixation Period, Monthly Interest Rate Survey (in years), Federal Housing Finance Agency (FHFA). Discontinued in 2019.



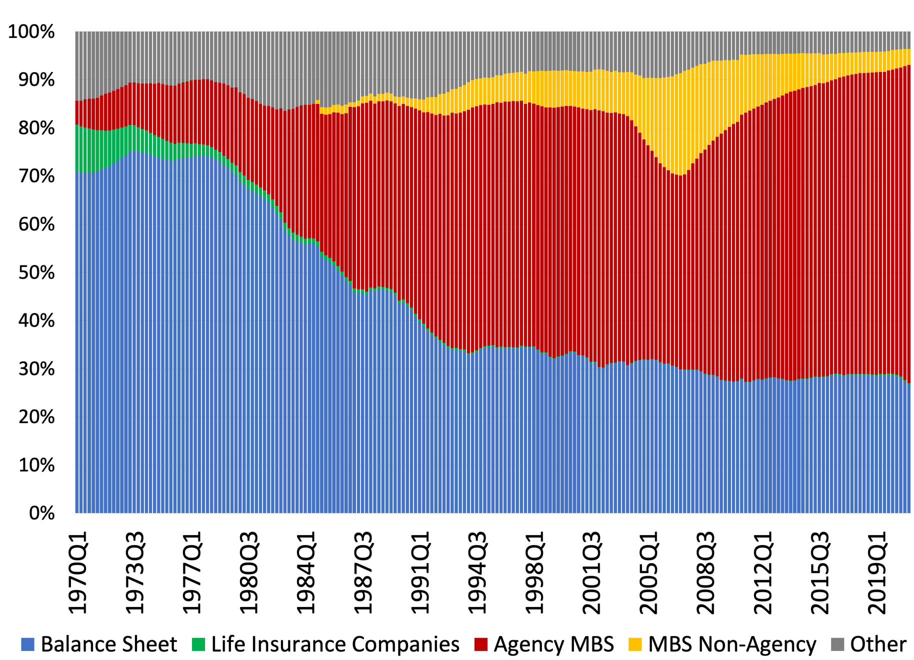
7.2 Mortgage Funding in the US

In this section, I describe how mortgages have been funded in the US since the 70s, as well as the role of Fannie Mae and Freddie Mac, and Ginnie Mae. At the end, I show some evidence that supports the relevance of these agencies in the mortgage market.

Figure 10 shows how the residential mortgage market is funded in the US[93]. For the mortgage-backed securities, the one-to-four-family residential mortgages that have been securitized and removed from the originator's balance sheet are reported as assets of either the sector for agency-backed mortgage pools or the sector for private issuers of asset-backed securities.

[93]One-to-four-family residential mortgages, also referred to as home mortgages, are loans collateralized by residential properties with one to four units and condominiums and cooperatives in structures with five or more units, as well as construction and land development loans on residential properties.

Figure 10: US Mortgage Funding of One-to-Four Family Residential Mortgages (% of Total Residential Mortgages Outstanding), Financial Accounts of the United States, Board of Governors of the Federal Reserve System.



7.2.1 Government-sponsored enterprises

The Government-sponsored enterprises (GSEs), Fannie Mae and Freddie Mac, buy single-family mortgages from mortgage companies, commercial banks, credit unions, and other financial institutions, It is worth mentioning that these mortgages *are not insured or guaranteed by a federal agency.* After buying the mortgages, they pool them to create MBSs, and sell the securities to investors. Fannie Mae and Freddie Mac guarantee the payment of principal and interest on their MBS and charges a fee for providing that guarantee. The guarantee fee (g-fee), covers projected credit losses from borrower defaults over the life of the loans, and some administrative costs.

Although during the 60s and 70s, Fannie Mae's and Freddie Mac's were established as privately-owned corporations, agency-MBS investors assumed (correctly) that the social costs threatened by the GSEs failure would compel a government bailout. In other words, even if the government's statutes toward the GSEs did not include an explicit guarantee, the market functioned assuming a functioning implicit guarantees. Private sector market participants believe the "tail risk" associated with a catastrophe falls on the government (Passmore and von Hafften, 2018). The reasoning behind this even before 2008, was explained by a range of benefits that resulted in lower operating and funding costs (Frame et al., 2004), such as a line-of-credit with the US Treasury, or previous stress episodes during the 90s in which the federal government assisted the troubled GSEs (Frame et al., 2015). Ultimately, the main difference between private mortgage insurance and government-backed insurance is that the former cannot avoid system-wide failure. As a result, GSE guarantees overshadow those produced in private markets. In particular, the agency-MBS trade in good and bad times, and therefore are able to carry a liquidity premium similar to Treasury securities. To some degree, my paper answers what would investors do in bad times when the GSEs are at risk of becoming insolvent and government support is uncertain.

Although Fannie Mae and Freddie Mac were originally independent entities, they have been under the government's control (in federal conservatorships) since the financial crisis of 2008. The Housing and Economic Recovery Act of 2008 (HERA), established the FHFA as their new regulator. As conservator, FHFA has the powers of the management, boards, and shareholders of Fannie Mae and Freddie Mac. Fannie Mae and Freddie Mac continue to operate as business corporations. Concurrently, the Treasury entered into senior preferred stock purchase agreements with each institution. Under these agreements, US taxpayers ultimately injected \$187.5 billion into Fannie Mae and Freddie Mac. In other words, the *implicit guarantee became explicit.* However the public support did not end there, some months after the conservatorship, the GSEs MBS markets faltered again. Liquidity was only reestablished after the Federal Reserve began purchasing these agency-MBS in November 2008[94].

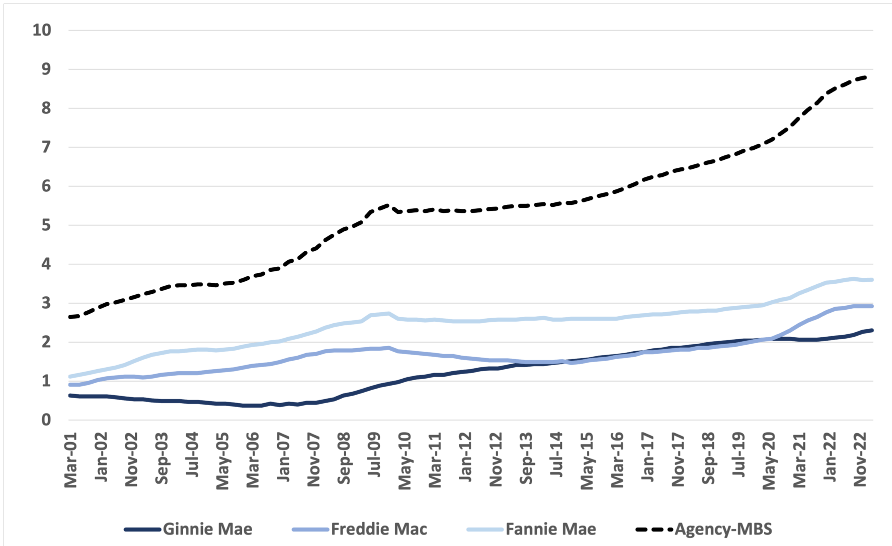
7.2.2 Ginnie Mae

This institution plays a similar role than that of Fannie Mae and Freddie Mac but for those mortgages that private financial institutions originate from residential mortgages that are *explicitly* insured or guaranteed by some federal program, such as those of the Federal Housing Administration (FHA), the Department of Veterans Affairs (VA), and the Department of Agriculture's Rural Housing Service (RHS). Unlike Fannie Mae and Freddie Mac, issuers of Ginnie Mae–guaranteed MBSs do not sell the underlying mortgages. Instead, the issuer, or a designated party, continues to receive payments from borrowers and forwards them (partially) to the MBS investors. Upon borrower's default, the issuer remove that borrower's loan from the MBS, repay the investor the remaining principal balance, and try to recover the amount owed from the borrower. After the foreclosure process, the issuer may settle any potential claim with the primary government guarantor, for example the FHA.

[94] Interestingly for this paper, the Federal Reserve's Large-Scale Asset Purchase (LSAP) program was targeted only to fixed-rate agency mortgage backed-securities between 2008 and 2010, see (Moench et al., 2010).

7.2.3 Agency Mortgage-Backed Securities Overview

Figure 11: Outstanding Agency Mortgage-Backed Securities (Trillion of Dollars), Global Market Analysis Report, Ginnie Mae.



In this section I briefly describe the size of the mortgage market and the relevance that the Agency MBS have on it. To put numbers in perspective, for the first quarter of 2023, the total housing market's value owned by households declined is \$41.2 trillion dollars, while outstanding mortgage debt owed by households equals \$12.5 trillion dollars[95] (i.e households' housing equity sits around \$28.7 trillion dollars) (Goodman et al., 2023). By the end of 2022, agency MBS accounted for 66.2 percent of the total mortgage debt outstanding, representing around \$8.9 trillion dollars. Unsecuritized first liens amount to \$3.7 trillion or 27.3 percent of the total mortgage debt outstanding[96], for the private-label securities these numbers are way smaller \$0.43 trillion dollars or 3.2 percent of total debt, see Figure 10.

By the end of the first quarter of 2023, out of the \$8.9 trillion of agency-MBS, 41 percent (\$3.6 trillion) was issued by Fannie Mae, 33.3 percent (\$2.9 trillion) was issued by Freddie Mac, and the remaining 25.8 percent (\$2.3 trillion) by Ginnie Mae, see Figure 11. Suggesting a equally important role for each of the three institutions on the US mortgage market. However, this was not always the case, during the housing boom in the early 2000s, Fannie Mae and Freddie Mac experienced fast-paced growth, in 2001 Fannie Mae and Freddie Mac MBS represented around 2 trillion dollars, by 2008 that had already doubled to 4.3 trillions dollars. However for that same period, Ginnie securitization volumes actually dropped representing less than 10% of

[95] The total public debt outstanding for the US in the first quarter on 2023 is \$ 32.8 trillion dollars.
[96] Bank's balance sheet: \$2.6 trillion, credit unions: \$0.56 trillion, other depository institutions \$0.54 trillion.

the agency-MBS outstanding in 2006. Ginnie Mae, in general, funds riskier mortgages. During the housing boom some of those borrowers might chose to substitute the FHA or VA loans for mortgages securitized by the private sector. After the mortgage market collapsed, the Private-label securities disappeared and Ginnie Mae started closing the gap by increasing it securitized volume (even securitizing, for a brief period of time during the mid 2010s, more than Freddie Mac itself). An interesting future research avenue is to understand the effect that the increasing role of Ginnie Mae on the mortgage market has on the housing market and the economy in general, for example, given that the government guarantee is in fact explicit for those agency-MBS, (Allen et al., 2022) study the effects of a segmented mortgage market with various sub-markets offering different credit standards, with particular emphasis on the Canadian economy.

7.2.4 Proposed Housing Finance Reforms

In the years following the 2008 financial crisis, a series of legislative proposals have sought to eliminate or substantially reduce the government credit guarantee. Initially after the crisis, the Obama Administration released a plan to limit the government's involvement in providing mortgage credit in February 2011[97]. Subsequently, the Housing Finance Reform and Taxpayer Protection Act of 2014 proposed that the GSEs stop offering guarantees on MBSs.[98] Instead, the Federal Mortgage Insurance Corporation (FMIC) would be created to provide an *explicit and partial federal guarantee* on principal and interest payments, similar to the guarantees provided by the GSEs, for MBSs while requiring private capital to absorb some losses before federal capital would step in[99]. The Congressional Budget Office (CBO) estimated at the time that under the bill, the FMIC would guarantee about 30 percent of the total mortgage market by 2024[100], while 60 percent would be supported by private firms with no federal guarantee and 10 percent would be guaranteed by other federal administrations.[101]. More recently, the Bipartisan Housing Finance Reform Act of 2018 proposed a larger restructuring of the mortgage market to increase the use of private capital (and not only for those MBSs issued by the GSEs). In addition to curtailing the GSEs' privileges, as the 2014 reform had proposed, they also proposed modifying the role that Ginnie Mae currently plays. As Ginnie Mae is the only association that explicitly provides a government guarantee on their MBSs, the proposal would

[97] "Reforming America's Housing Finance Market: A Report to Congress", US Department of Treasury and US Department of Housing and Urban Development, February 2011.

[98] Housing Finance Reform and Taxpayer Protection Act of 2014, S. 1217, 113$^{\text{th}}$ Congress, 2$^{\text{nd}}$ Session, 2014.

[99] "Congressional Budget Office Cost Estimate of S. 1217 Housing Finance Reform and Taxpayer Protection Act of 2014," Congressional Budget Office, September 5, 2014.

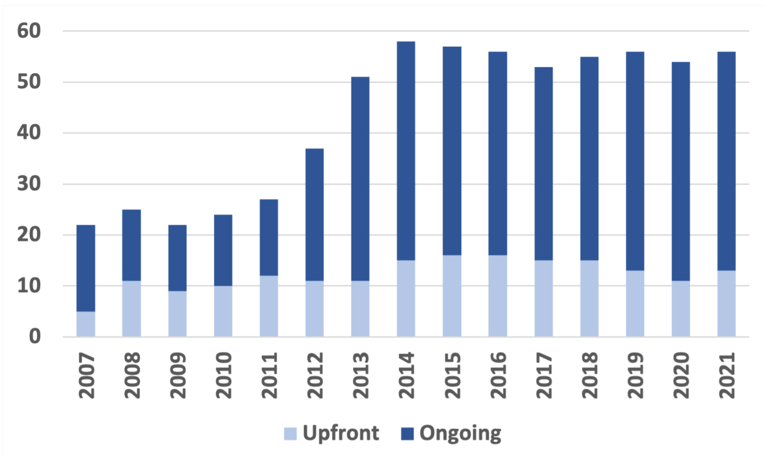
[100] Ibid.

[101] Ibid.

allow Ginnie Mae to (1) keep providing a guarantee on MBSs, (2) establish a new program called Ginnie Mae Plus that would set qualifications for insurable mortgages, and (3) grant Ginnie Mae the authority to guarantee private insurance-backed MBSs if the mortgages meet qualifications and the insurance is provided by eligible Private Credit Enhancers (PCEs).[102] Additionally, the reform proposed a not-for-profit Mortgage Security Market Exchange to develop common standards "for the private securitizing, pooling, and servicing of mortgages"[103]. If any of these legislative proposals were accepted, they would turn a largely public housing finance market into a largely private one.

7.3 Guarantee Fee

Figure 12: Average Guarantee Fee charged by Fannie Mae and Freddie Mac (Basis Points), Fannie Mae and Freddie Mac Single-Family Guarantee Fees, FHFA Reports.



The guarantee fee (g-fee) is intended to cover the credit risk and other administrative and operational costs that the GSEs incur when they acquire single-family loans from sellers. There are two types of guarantee fees: ongoing and upfront. Ongoing fees are factored into each loan's interest rate and collected each month over the life of a loan. Upfront fees are one-time payments made by sellers upon loan delivery to any of the GSEs[104](FHFA, 2020). Figure 12, shows the average g-fee charged by the GSES presenting the upfront fees in annualized form.

The larger modifications in the g-fee occurred after the housing and mortgage markets collapsed. The first of these modifications occurred in March 2008, and the main goal has always been to better price credit risk.

[102] "The Bipartisan Housing Finance Reform Act Summary of Key Provisions: Overview," Financial Services Committee, House of Representatives.

[103] Ibid, page 5.

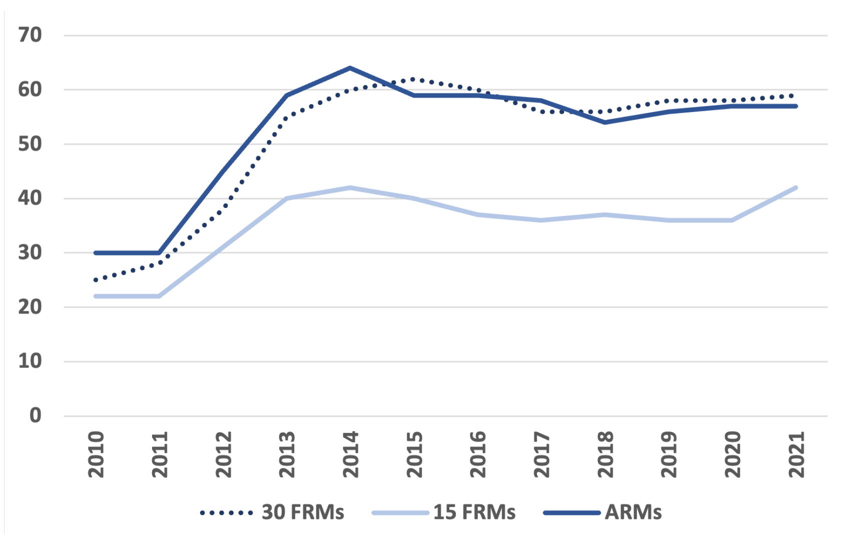
[104] Ongoing fees are based primarily on the product type, such as whether the loan is a 30-year fixed rate or a 15-year fixed rate loan. Upfront fees are based primarily on specific risk attributes.

Specifically, the FHFA as conservator required increases in g-fees related to risk factors such as product-type or borrower's LTV ratio, credit score and mortgage purpose[105]. Figure 12 shows how in subsequent years after 2008 the GSEs gradually raised the g-fees from 22 bps to 51 bps in 2013. After 2013, although some changes have happened, the g-fee has not moved substantially going from 51 bps in 2013 to 56 bps in 2021.

7.3.1 Guarantee fees on ARMs

In this section I show the average guarantee fee by product type. 30-year FRMs, 15-year FRMs, and ARMs. The housing market collapse forced the GSE to implement a guarantee fee increase in March 2008 to better price credit risk. Specifically, they increased the ongoing fees and introduced two new upfront fees: a fee based on a borrower's LTV ratio and credit score. After this first increase, the conservator FHFA directed the GSEs to increase their guarantee fees by an additional 10 basis points to protect taxpayers further. The increase was allocated to longer-term mortgages than for example the shorter 15 years loans to better align the term risks, see Figure 13 where I show the average g-fee by Product Type. It is worth mentioning that the product type is referenced in the ongoing fraction of the g-fee (see Figure 12).

Figure 13: Average Guarantee Fee charged by Fannie Mae and Freddie Mac by Product Type (Basis Points), Fannie Mae and Freddie Mac Single-Family Guarantee Fees, FHFA Reports.



In Figure 14, I zoom on the last years and show the guarantee fee by product type from 2018-2021. As described before, the guarantee fees on 15-year FRMs is around 15 bps lower than that of 30-year FRMs and ARMs. This is intended the price the product risk. However, in Figure 15, I show what is known as the *average gap*. Shortly, the gap measures the charged guarantee fee minus the costs and the target rate of

[105]Purchase, cash-out refinancing or rate refinancing.

return, that is an measurement of the risk-adjusted profitability (by product type) on new loan acquisitions. For 30-year FRMs this measure is very close to zero in all years, indicating that the guarantee-fee charged is enough to cover the mortgage losses (notice these are years where the economy did not suffer any housing crisis). However, for ARMs this measurement is always above 10 bps, this indicates that the guarantee-fee is effectively so large that it exceeds the mortgage losses observed for ARMs. A similar observation holds for 15-year FRMs. This could be suggestive evidence that the GSEs effectively subsidize the securitization of 30-year FRMs with other mortgage products.

Figure 14: Average Guarantee Fee charged by Fannie Mae and Freddie Mac by Product Type (Basis Points), Fannie Mae and Freddie Mac Single-Family Guarantee Fees, FHFA Reports.

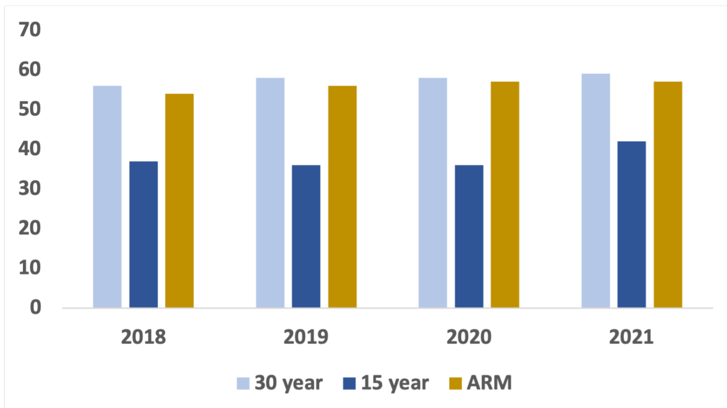
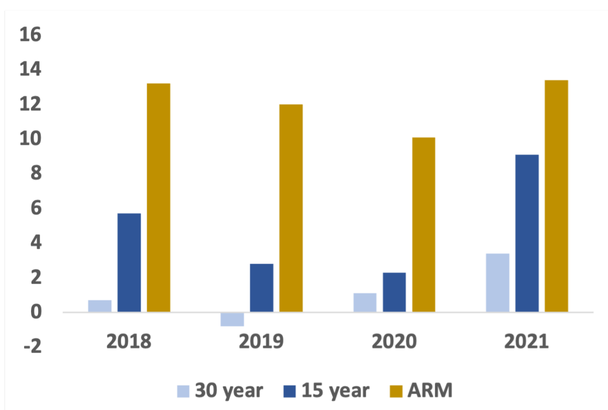


Figure 15: Average Gap on Fannie Mae and Freddie Mac securitized mortgages by Product Type (Basis Points), Fannie Mae and Freddie Mac Single-Family Guarantee Fees, FHFA Reports.



7.4 Mortgage Market in the UK

In this section, I present some motivating facts for the UK mortgage market. First, I show that in the UK mortgages have typically short-terms (relative to the US). Second, I show that mortgage funding comes mainly through the balance sheet of depository institutions. The UK mortgage market is the largest mortgage market in Europe, see section 7.5.1. In Figure 21, I show that the homeownership rate and the share of homeowners with a mortgage is similar to the US.

7.4.1 Mortgage Choice in the UK

Figure 16 and Figure 17 show the mortgage outstanding and mortgage originations, respectively, by its initial fixation length period. The 4 categories are: (i) from 0 - 1 year of fixation period, (ii) from 1 - 5 years of fixation periods, (iii) from 5 - 10 years of fixation periods, and (iv) over 10 years. The originations and mortgage outstanding in the UK are mainly composed of short-term debt (relative to the US), the 1 - 5 years bracket accounts for the majority of mortgages.

Figure 16: Residential Mortgage Outstanding for the UK, Bank of England

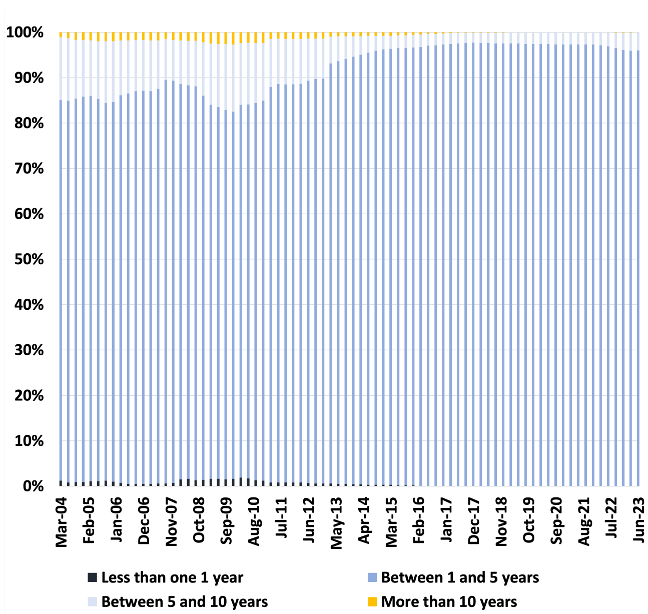
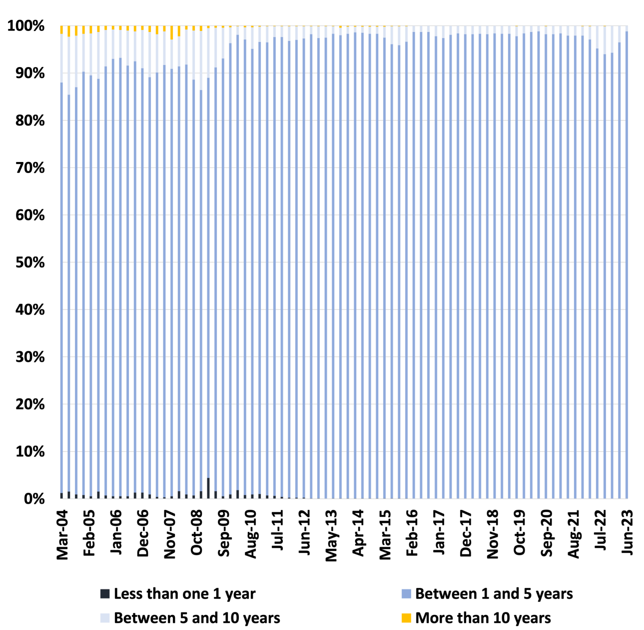


Figure 17: Residential Mortgage Originations for the UK, Bank of England



In Figures 18 and 19, I plot the mortgage outstanding and mortgage originations, respectively, by its initial fixation length period with greater detail in the short-term brackets. The 3 categories are: (i) from 0 - 2 year of fixation period, (ii) from 3 - 4 years of fixation periods, (iii) over 5 years of fixation period. The information provided by in these plots shows a substitution from very short initial fixation period mortgages (2-4 years) to slightly longer initial fixation period mortgages (more than 5 years), which is currently reverting given the rate increases after the covid pandemic. The increase in the over 5 year fixation period mortgages are actually due to mortgages with an *exact* initial fixation period of 5 years (see Figures 16 and Figure 17 or (Cloyne et al., 2019) where they explain that around 90% of the mortgage outstanding in the UK are due to the 2-year and 5-year mortgages).

7.4.2 Mortgage Funding in the UK

Figure 20 shows how the residential mortgage market is funded in the UK. The plot shows the fraction of the UK's mortgage outstanding for each year that is funded by each of the categories. For the case of the UK, the main source of funding comes from retail deposits and wholesale funding. It is also worth mentioning that there are no subsidies which apply to house purchase on the whole (as in the US), there are however some subsidies for specific parts of the market, such as those who live in social housing (EMF, 2022). Therefore, and relevant to this paper, all the mortgage risks (credit, prepayment, and interest rate

Figure 18: Residential Mortgage Funding for the UK, Bank of England

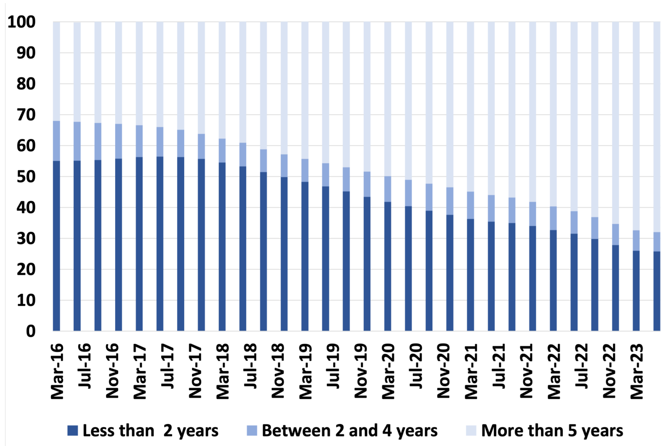
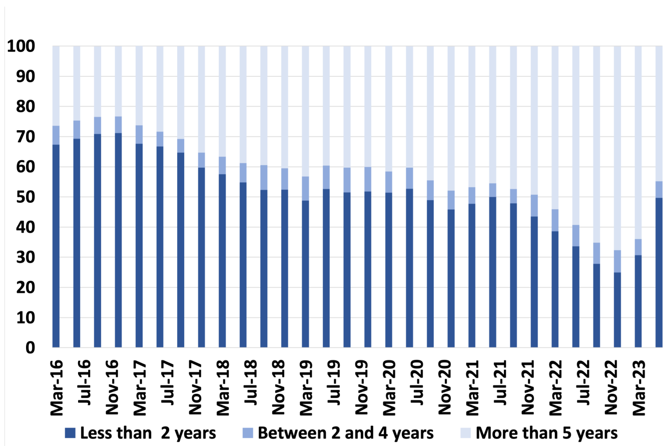
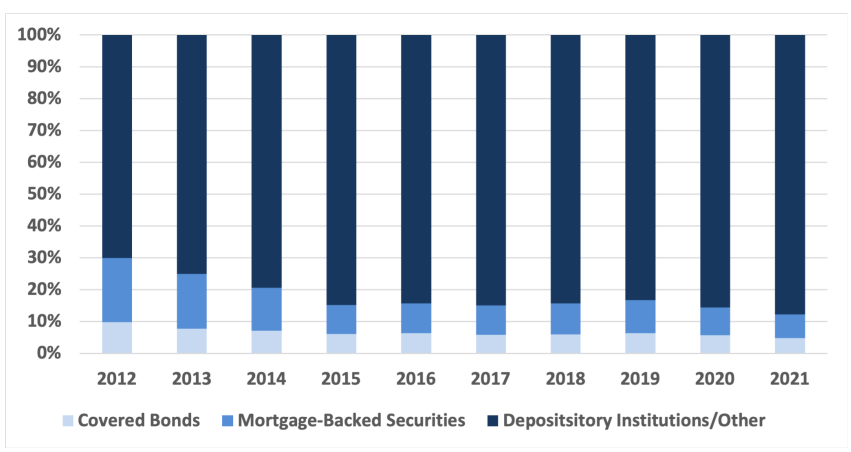


Figure 19: Residential Mortgage Funding for the UK, Bank of England



risks) are borne by the financial intermediary in the UK.

Figure 20: Residential Mortgage Funding for the UK, Bank of England



7.5 Mortgage Markets in Europe

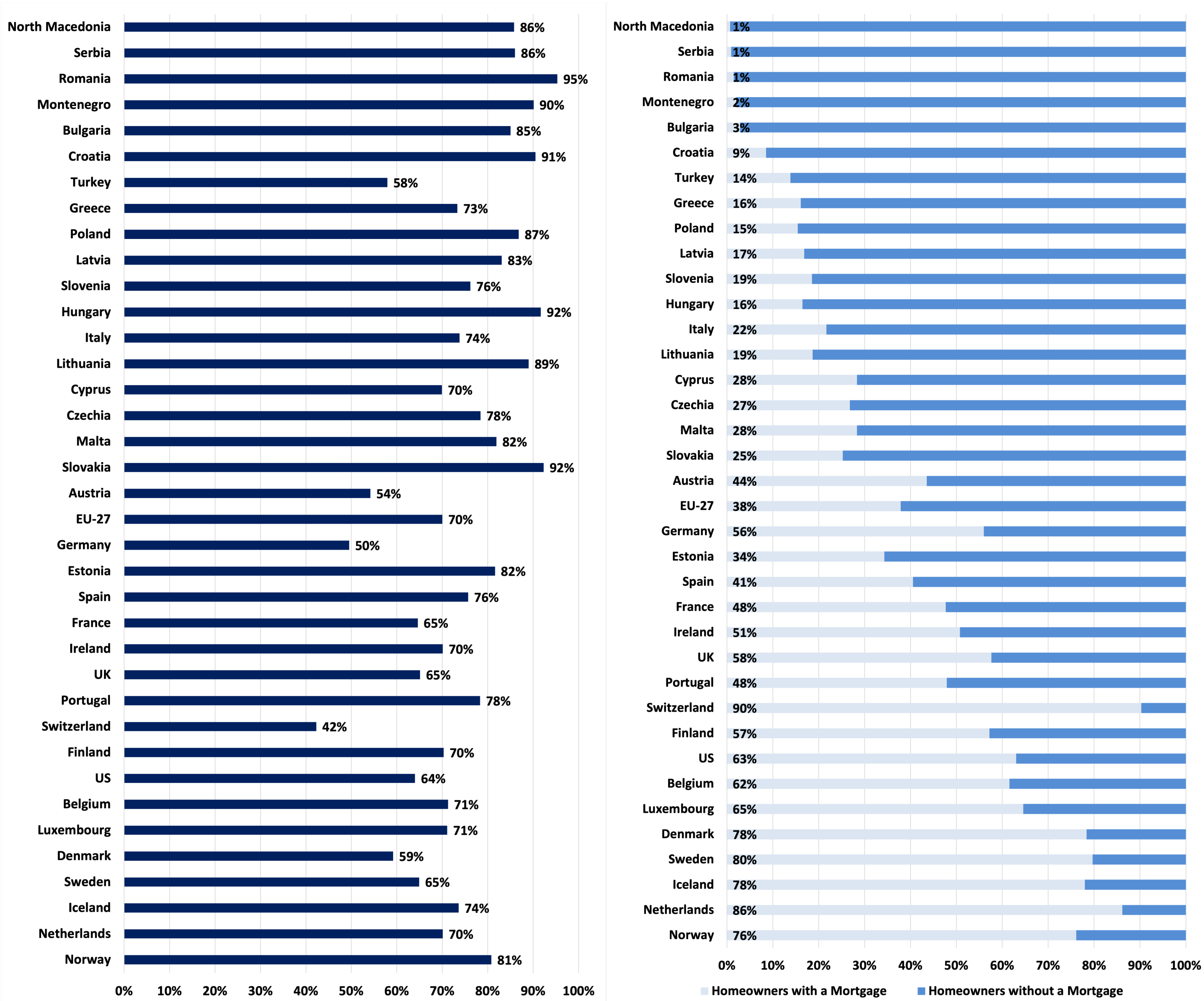
7.5.1 Overview of the Mortgage Markets

The total value of the EU mortgage market was 6.5 trillion euros in 2021. Adding the UK, Norway and Iceland, the figure reached 8.7 trillion euros outstanding. The US mortgage market was valued at 11.9 trillion dollars (Federal Reserve, 2023) in 2022. Germany, France, the Netherlands, Spain, Sweden and the UK combined represent more than 75% of outstanding mortgages in the EU27 and UK aggregate. The UK and German markets in 2021 were the largest, at 1.8 trillion euros and 1.7 trillion euros respectively. Figure 21 presents the homeownership rates and the share of homeowners that hold a mortgage for a set of European countries and the US. The US does not stand out in any of these two measures.

Figure 21: Homeowner Share and Mortgage Holders in 2021, EMF Hypostat 2022

(a) Home Ownership Rates

(b) Share of Homeowners with a Mortgage



7.5.2 Mortgage Choice in Europe

Figure 22 shows the share of mortgages by fixation period for various European countries. The dataset used is taken from the EMF Hypostat 2022[106]. The 4 categories of initial fixation periods are: (i) from 0 - 1 year of fixation period, (ii) from 1 - 5 years of fixation periods, (iii) from 5 - 10 years of fixation periods, and (iv) over 10 years (i.e this bracked includes 15-year and 30-year fixed-rate mortgages). Figure 23 shows an

[106]The EMF-ECBC's flagship statistical report presents data on recent developments in housing and mortgage markets in Europe.

alternative (but slightly similar) definition for the ARM share. It shows the average initial fixation period of mortgages outstanding for a variety of different economies, here I do include the US[107]. The dataset is taken from the public dataset presented in (Campbell and Cocco, 2015). Both measures show that it is not common to see economies that hold a majority of long-term contracts that fix the interest rate for the entire term (such as 30-year FRMs), expect the US.

Figure 22: Mortgage Market Gross Issuance breakdown by Interest Rate Type in 2021, EMF Hypostat 2022

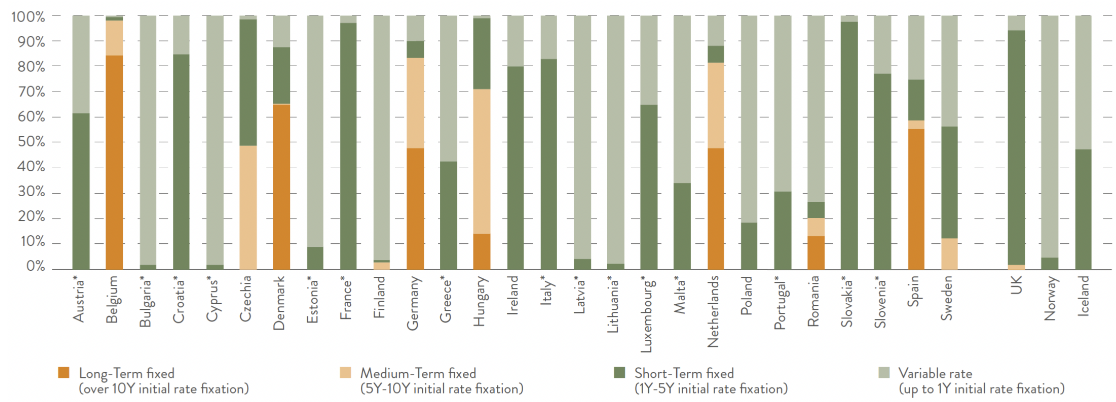
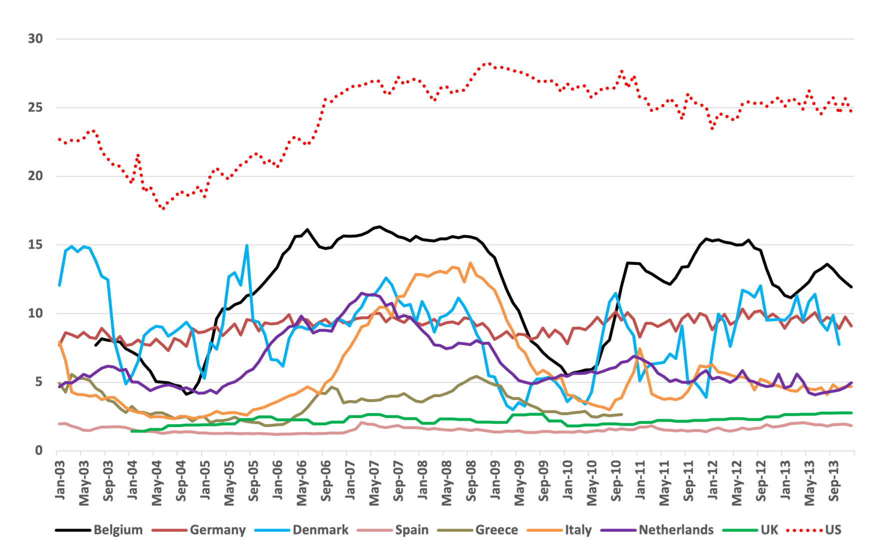


Figure 23: Average Initial Fixation Period (years), taken from (Campbell and Cocco, 2015)



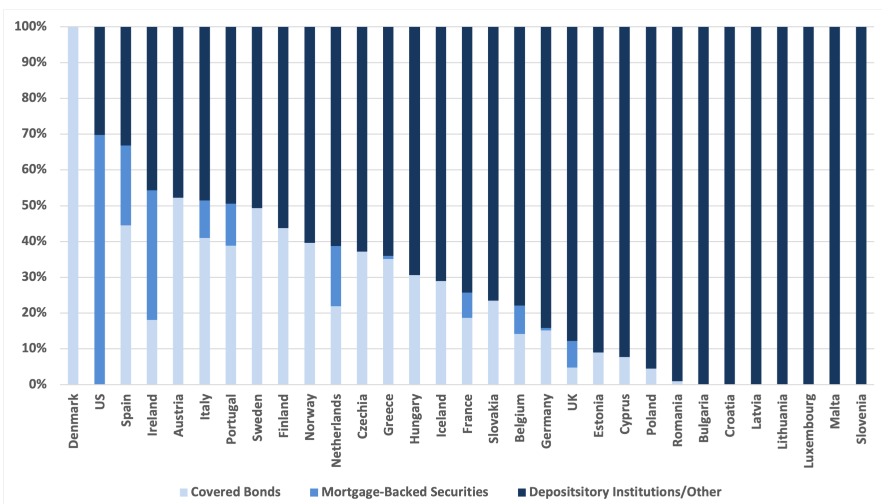
7.5.3 Mortgage Funding in Europe

Figure 24 shows how the residential mortgage markets are funded in various European countries. For each country, the plot shows the fraction of the mortgage outstanding that is funded by each of the categories as

[107] A longer time series for the US is plotted in Figure 9.

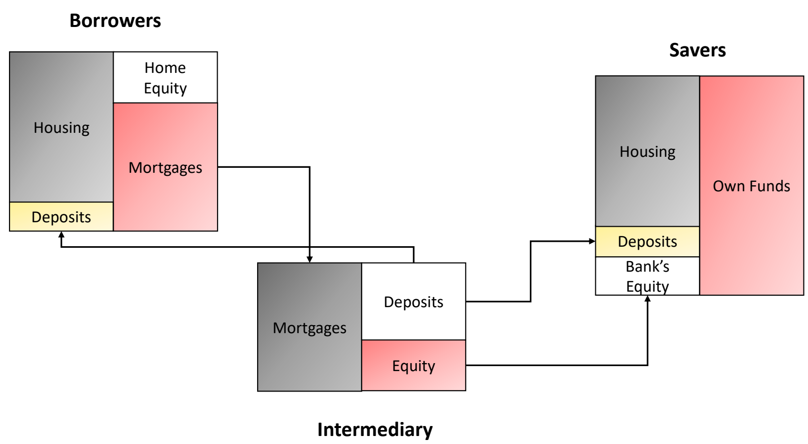
of 2022. The first observation is that mortgage-backed securities are not as close as popular as in the US for any of the countries presented here. The second observation, however, is that some mortgage market are able to issue long-term financing through other sources such as covered bonds. Covered bonds – debt instruments secured by a pool of assets – have been an overwhelmingly European phenomenon for most of their 250-year history. Lately, they have gown in popularity in new markets such as Australia, Canada, and Singapore. According to the European Mortgage Federation, covered bonds finance around 25% of European mortgage loans. Issuers of these bonds like the ability to diversify their funding sources away from short-term deposits and toward longer-term instruments, which better match the maturity of their mortgage book (EMF, 2022). Denmark is an outstanding example of a market that heavily relies on covered bonds and successfully originates a large fraction of FRMs. The relative benefits of covered bonds with respect to other sources of funding, such as securitization, and its effectiveness in maintaining a certain type of mortgage structure is a potential future research avenue. See for example, (Carbó-Valverde et al., 2017; Correia and Pinto, 2023).

Figure 24: Residential Mortgage Funding (Country Comparison, 2021), EMF Hypostat 2022



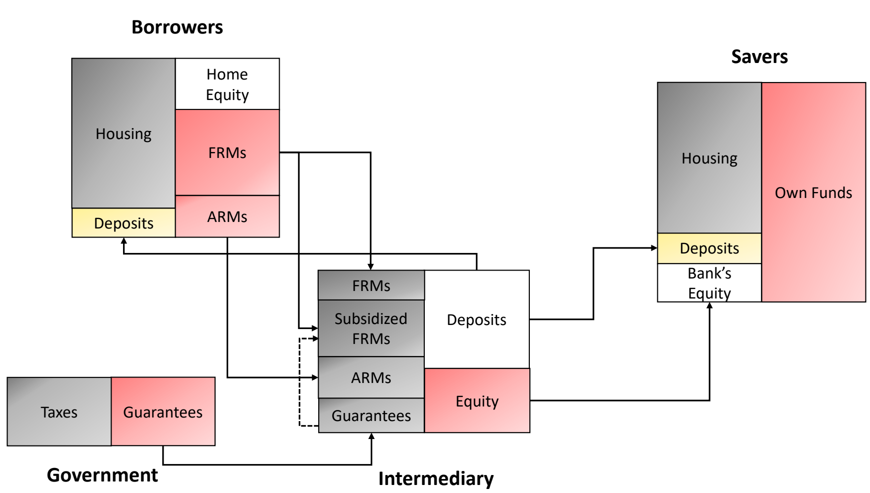
7.6 Model Overview

Figure 25: Balance sheets of agents in the model economy, traditional financial system



Traditional financial system. Banks hold mortgages as assets and issue equity and debt. Savers indirectly fund borrowers by holding all equity and deposits. Borrowers use housing as collateral, and only have the option of holding FRMs.

Figure 26: Balance sheets of agents in the model economy, modern financial system with mortgage choice



Financial system with mortgage choice and credit guarantees. Banks insure against credit risk a fraction of their FRMs. Borrowers use housing as collateral. Borrowers are able to fund its house purchase with either FRMs or ARMs. The government provides insurance for those mortgages that the bank decides to insure.

7.7 Recursive Representation for Aggregate Mortgage Payments

In this section, I describe the mortgage payments that a borrower needs to make under different status based on (i) the mortgage choice at origination, and (ii) whether the mortgage was recently originated (refinanced) or not. Some useful notation is in order. Variables with a * superscript (x^*) refers to the choice variables coming from those borrowers with newly originated mortgages, variables without a * superscript (x) refer to choice variables coming from borrowers that did not originated a mortgage in the previous period.

Mortgage Payments for Refinancers In this section I describe the mortgage payments due on mortgages that were originated in period t that are due in period $t+1$.

FRMs A mortgagor that refinances its FRM to $m_t^{frm,*}$ in period t, will have a payment due in period $t+1$ equal to the interest rate chosen at period t from the menu of FRM contract[108] as described in (2) multiplied by the new refinanced mortgage outstanding plus the principal payment due in period $t+1$,

$$\text{payment}_{t+1}^{frm,*} = \underbrace{\iota^{frm}(\alpha_t^{B,*}, \mathcal{Z}_t) \cdot m_t^{frm,*}}_{\text{Interest Payment}} + \underbrace{(1-\delta) \cdot m_t^{frm,*}}_{\text{Principal Payment}} \tag{65}$$

ARMs On the other hand, a mortgagor that refinances its ARM to $m_t^{arm,*}$ in period t, the total mortgage payment at period $t+1$ for this refinanced mortgage equals the interest rate chosen at period t from the menu of ARM contract[109] as described in (3) multiplied by the new refinanced mortgage outstanding plus the principal payment due in period $t+1$,

$$\begin{aligned}\text{payment}_{t+1}^{arm,*} &= \underbrace{\iota^{arm}(\alpha_t^{B,*}, \mathcal{Z}_t, \mathcal{Z}_{t+1}) \cdot m_t^{arm,*}}_{\text{Interest Payment}} + \underbrace{(1-\delta) \cdot m_t^{arm,*}}_{\text{Principal Payment}} \\ &= \Big(spread^{arm}(\alpha_t^{B,*}, \mathcal{Z}_t) + i(\mathcal{Z}_{t+1}) + (1-\delta)\Big) \cdot m_t^{arm,*} \end{aligned} \tag{66}$$

Mortgage Payments for Non-Refinancers The previous description on mortgage payments only applies to newly originated mortgages, i.e. mortgages originated in period t whose first payment is due in period $t+1$. However, my model features long-term mortgages. In this section I describe the mortgage payments due on mortgages that were originated in some period $\tau < t$, and due in period $t+1$.

[108] Notice that the menu contract depends on the set of choices for those borrowers with new originations in period t, $\alpha_t^{B,*}$. Hence, I explicitly express the interest rate function as $\iota^{frm}(\alpha_t^{B,*}, \mathcal{Z}_t)$. For a complete description of $\alpha_t^{B,*}$, see section 3.3

[109] Notice that the menu contract depends on the set of choices for those borrowers with new originations in period t, $\alpha_t^{B,*}$. Hence, I explicitly express the spread function as $spread^{arm}(\alpha_t^{B,*}, \mathcal{Z}_t)$. For a complete description of $\alpha_t^{B,*}$, see section 3.3.

FRMs The timing of the model is widely discussed in section 3.4. However, it is worth mentioning that at period t, a borrower holding a FRM knows exactly its payment due in period $t+1$ in the case it did not refinanced or defaulted on any period prior to t. To be more specific, at period t, a borrower that originated a FRM in any period $\tau < t$ contracted a promise to pay the following amount in period $t+1$,

$$\text{payment}_{t+1}^{frm,\tau} = \delta^{t-\tau}\left[\iota^{frm}(\alpha_\tau^{B,*}, \mathcal{Z}_\tau) \cdot m_\tau^{frm,*} + (1-\delta) \cdot m_\tau^{frm,*}\right] \tag{67}$$

where $m_\tau^{frm,*}$ is the mortgage debt newly originated in period τ, and $\iota^{frm}(\alpha_\tau^{B,*}, \mathcal{Z}_\tau)$ is the FRM contract function defined in (2) and evaluated at the financial portfolio chosen at τ, $\alpha_\tau^{B,*}$, and at the aggregate state in that same period, $\mathcal{Z}_\tau$. Notice that the right hand side of this expression *does not* depend on the payment period (in this case $t+1$) since by definition the payments on a FRM should not depend on it, I just write $\text{payment}_{t+1}^{frm,\tau}$ for completeness. The decaying parameter δ affects the whole expression, the assumption I make here is that the mortgage outstanding and the mortgage interest payments decay at the same rate.

Consider a sequence of periods $T = \{t-a, \dots, t-1, t\}$ for some arbitary $a < t$, and a sequence of portfolio choice sets $\{\alpha_\tau^{B,*}\}_{\tau\in T}$, which implies a sequence of FRM originations $\{m_\tau^{frm,*}\}_{\tau\in T}$. For this Appendix, the originations can come from heterogeneous borrowers (although in the main text we use the aggregation result). Finally, assume no default or prepayment only for this section.

The **aggregate payment** (AP) to an intermediary from all mortgage vintages in T equals,

$$AP_{t+1}^{frm} = \sum_{\tau\in T} \text{payment}_{t+1}^{frm,\tau} \tag{68}$$

where I implicitly assume that $\tau = t$ represents the variables for those newly originated mortgages, instead of using the * notation as in (65). We can rewrite (68) as,

$$AP_{t+1}^{frm} = \sum_{\tau\in T} \text{payment}_{t+1}^{frm,\tau} = \sum_{\tau\in T} \delta^{t-\tau}\left[\iota^{frm}(\alpha_\tau^{B,*}, \mathcal{Z}_\tau) + (1-\delta)\right] \cdot m_\tau^{frm,*}$$

Define M_t^{frm} as the *aggregate FRM debt outstanding* in period t for those mortgages originated in periods prior and including t. Formally,

$$M_t^{frm} = \sum_{\tau\in T} \delta^{t-\tau} \cdot m_t^{frm,*}$$

We can define a recursive equations for M_t^{frm}.

$$\begin{aligned} M_{t-a}^{frm} &= m_{t-a}^{frm,*} \\ M_t^{frm} &= m_t^{frm,*} + \delta \cdot M_{t-1}^{frm}, \quad \forall t \end{aligned} \tag{69}$$

Similarly, define $M_t^{I,frm}$ as the *aggregate FRM interest payments* in period t for those mortgages originated in periods prior and including t. Formally,

$$M_t^{I,frm} = \sum_{\tau \in T} \delta^{t-\tau} \left[\iota^{frm}(\alpha_t^{B,*}, \mathcal{Z}_t) \cdot m_t^{frm,*} \right]$$

We can define a recursive equation for $M_t^{I,frm}$.

$$\begin{aligned} M_{t-a}^{I,frm} &= \iota^{frm}(\alpha_{t-a}^{B,*}, \mathcal{Z}_{t-a}) \cdot m_{t-a}^{frm,*} \\ M_t^{I,frm} &= \iota^{frm}(\alpha_t^{B,*}, \mathcal{Z}_t) \cdot m_t^{frm,*} + \delta \cdot M_{t-1}^{I,frm}, \quad \forall t \end{aligned} \tag{70}$$

Using (69) and (70) we can fully characterize the aggregate FRM payment to the intermediary (68) as,

$$AP_{t+1}^{frm} = M_t^{I,frm} + (1-\delta) \cdot M_t^{frm}$$

This equation shows how to compute recursive equations for the aggregate mortgage payments coming from FRMs (i.e. coming from different mortgage vintages). Those equations will be the first step to computing the the law motions for mortgage outstanding and mortgage interest payments that arise from the borrower aggregation result detailed in section 3.5.4.

ARMs On the other hand, in period t, a borrower holding an ARM only partially knows what its mortgage payment in period $t+1$ will look like, even if she did not refinanced or defaulted on any period prior to t. To be more specific, at period t, a borrower that originated an ARM in any period $\tau < t$ contracted a promise to pay the following amount in period $t+1$,

$$\begin{aligned} \text{payment}_{t+1}^{arm,\tau} &= \delta^{t-\tau} \left[\iota^{arm}(\alpha_\tau^{B,*}, \mathcal{Z}_\tau, \mathcal{Z}_{t+1}) \cdot m_\tau^{arm,*} + (1-\delta) \cdot m_\tau^{arm,*} \right] \\ &= \delta^{t-\tau} \cdot \big(spread^{arm}(\alpha_\tau^{B,*}, \mathcal{Z}_\tau) + i(\mathcal{Z}_{t+1}) + (1-\delta) \big) \cdot m_\tau^{arm,*} \end{aligned} \tag{71}$$

where $m_\tau^{arm,*}$ is the mortgage debt newly originated in period τ, and $spread^{arm}(\alpha_\tau^{B,*}, \mathcal{Z}_\tau)$ is the ARM spread contract function defined in (3) and evaluated at the financial portfolio chosen at τ, $\alpha_\tau^{B,*}$, and at the aggregate state in that same period, $\mathcal{Z}_\tau$. Notice that the right hand side of this expression *does* depend on the payment period (in this case $t+1$) since by definition the payments on an ARM depends on the current's period policy rate, regardless of the origination period τ. The decaying parameter δ affects the whole expression, the assumption I make here is that the mortgage outstanding and the mortgage interest payments decay at the same rate as with the FRM.

Again, consider a sequence of periods $T = \{t-a, \ldots, t-1, t\}$ for some arbitary $a < t$, and a sequence of portfolio choice sets $\{\alpha_\tau^{B,*}\}_{\tau \in T}$, which implies a sequence of ARM originations $\{m_\tau^{frm,*}\}_{\tau \in T}$. For this Appendix, the originations can come from heterogeneous borrowers (although in the main text we use the aggregation result). Finally, assume no default or prepayment only for this section.

The **aggregate payment** (AP) for ARMs to an intermediary coming from all mortgage vintages in T,

$$AP_{t+1}^{arm} = \sum_{\tau \in T} \text{payment}_{t+1}^{arm,\tau} \tag{72}$$

where I implicitly assume that $\tau = t$ represents the variables for those newly originated mortgages, instead of using the * notation as in (66). We can rewrite (72) as,

$$\begin{aligned} AP_{t+1}^{arm} &= \sum_{\tau \in T} \text{payment}_{t+1}^{arm,\tau} = \sum_{\tau \in T} \delta^{t-\tau} \left[\iota^{arm}(\alpha_\tau^{B,*}, \mathcal{Z}_\tau, \mathcal{Z}_{t+1}) + (1-\delta)\right] \cdot m_\tau^{arm,*} \\ &= \sum_{\tau \in T} \delta^{t-\tau} \left(spread^{arm}(\alpha_\tau^{B,*}, \mathcal{Z}_t) + i(\mathcal{Z}_{t+1}) + (1-\delta)\right) \cdot m_\tau^{arm,*} \end{aligned}$$

Define M_t^{arm} as the *aggregate ARM debt outstanding* in period t for those mortgages originated in periods prior and including t. Formally,

$$M_t^{arm} = \sum_{\tau \in T} \delta^{t-\tau} \cdot m_t^{arm,*}$$

We can define a recursive equations for M_t^{arm}.

$$\begin{aligned} M_{t-a}^{arm} &= m_{t-a}^{arm,*} \\ M_t^{arm} &= m_t^{arm,*} + \delta \cdot M_{t-1}^{arm}, \quad \forall t \end{aligned} \tag{73}$$

Similarly, define $M_t^{S,arm}$ as the *aggregate ARM spread-interest payments* in period t for those mortgages originated in periods prior and including t. Formally,

$$M_t^{S,arm} = \sum_{\tau \in T} \delta^{t-\tau} \left[spread^{arm}(\alpha_t^{B,*}, \mathcal{Z}_t) \cdot m_t^{arm,*} \right]$$

We can define a recursive equation for $M_t^{S,arm}$.

$$\begin{aligned} M_{t-a}^{S,arm} &= spread^{arm}(\alpha_{t-a}^{B,*}, \mathcal{Z}_{t-a}) \cdot m_{t-a}^{arm,*} \\ M_t^{S,arm} &= spread^{arm}(\alpha_t^{B,*}, \mathcal{Z}_t) \cdot m_t^{arm,*} + \delta \cdot M_{t-1}^{S,arm}, \quad \forall t \end{aligned} \tag{74}$$

Using (73) and (74) we can fully characterize the aggregate ARM payment to the intermediary (72) as,

$$AP_{t+1}^{arm} = M_t^{S,arm} + [i(\mathcal{Z}_{t+1}) + (1-\delta)] \cdot M_t^{arm}$$

Notice that the equation for AP_{t+1}^{arm} is slightly different from the the one for FRMs, AP_{t+1}^{frm}. The variable interest term becomes recursive because of the mortgage outstanding equation (73) and not because of the spread-interest recursive equation (74).

As with the FRM, I show how to compute recursive equations for the aggregate mortgage payments coming from ARMs (i.e. coming from different mortgage vintages). Those equations will be the first step to computing the the law motions for mortgage outstanding and mortgage interest payments that arise from the borrower aggregation result detailed in section 3.5.4.

7.8 Borrower's Mortgage Pricing at the Trading Stage

Why are the pair of prices $(p_{t+1}^{B,m,j}, p_{t+1}^{B,I,j})$ for each $j \in \{arm, frm\}$ needed? Because (i) the trading stage requires mortgage balances to be priced $(p_{t+1}^{B,m,j})$ at any state z_{t+1}, and (ii) mortgage payments not only depend on the mortgage outstanding but also on the interest/spread rate at origination, hence mortgage payment need to be priced separately $(p_{t+1}^{B,I,j})$. The following example clarifies,

In the first example (Figure 27), a FRM is originated in period 1. The mortgage balance at origination is 10, the amortization schedule is such that $\delta = 1 - \frac{1}{30} = 0.97$, and the interest rate is 5%. The model's mortgage payment equals: $(1-\delta) \cdot m + i \cdot m = 0.33 * 10 + 0.05 * 10 = 0.83$[110].

[110] As a reference, the mortgage payment on a *real-life* 30-year FRM equals 0.65.

Figure 27: Mortgage schedule 1

$t = 1$	$t = 2$	$t = 3$
$m = 10$	$m = \delta \cdot 10$	$m = \delta^2 \cdot 10$
$i = 5\%$	$i = 5\%$	$i = 5\%$
$p = 0.83$	$p = 0.83$	$p = 0.83$
$p_{30} = 0.65$	$p_{30} = 0.65$	$p_{30} = 0.65$

Figure 28: Mortgage schedule 2

$t = 1$	$t = 2$	$t = 3$
	$m = 12$	$m = \delta \cdot 12$
	$i = 2\%$	$i = 2\%$
	$p = 0.64$	$p = 0.64$
	$p_{30} = 0.54$	$p_{30} = 0.54$

In the second example (Figure 28), a FRM is originated in period 2. The mortgage balance at origination is 12, the amortization schedule is such that $\delta = 1 - \frac{1}{30} = 0.97$, and the interest rate is 2% (interest rates decreases from one period to another). The model's mortgage payment in that case is equal to $(1 - \delta) \cdot m + i \cdot m = 0.333 * 12 + 0.02 * 12 = 0.64$[111]. This means the mortgage loan is **larger**, but the mortgage payment is substantially **smaller**. Therefore, in the trading stage in period 3 when these two borrowers re-balance their mortgage assets and merge into a single representative borrower it would not be enough to price only the history of mortgage only-principal payment assets ($p_{t+1}^{B,m,j}$), but I also need to price the history of mortgage only-interest payment assets ($p_{t+1}^{B,I,j}$).

[111] As a reference, the mortgage payment on a *real-life* 30-year FRM equals 0.54.

7.9 Borrower's Problem

Problem Definition Substituting the definition of the value function for refinancers (19) and for non-refinancers (20), the borrower's problem (22) in the main text can be written as,

$$
\begin{aligned}
V^B(w_t^B, \mathcal{Z}_t) = & \max_{\{c_t^B, s_t^B, \alpha_t^{B,\overline{nr}}, \alpha_t^{B,\overline{r}}\}} \frac{\left(\left(c_t^B\right)^{\theta^B}\left(s_t^B\right)^{1-\theta^B}\right)^{1-\gamma}}{1-\gamma} + \psi^B \cdot \frac{(d_t^{B,nr})^{1-\gamma}}{1-\gamma} \\
& + \sum_{l=\{r,nr\}} \beta^B \cdot q^l \cdot E_t\Bigg[\max\Bigg(V^B\left(w_{t+1}^{B,nd,l}, \mathcal{Z}_{t+1}\right), V^B\left(w_{t+1}^{B,(d,arm),l}, \mathcal{Z}_{t+1}\right), \\
& \qquad V^B\left(w_{t+1}^{B,(d,frm),l}, \mathcal{Z}_{t+1}\right), V^B\left(w_{t+1}^{B,d,l}, \mathcal{Z}_{t+1}\right)\Bigg)\Bigg]
\end{aligned}
\tag{75}
$$

subject to the budget constraint at the trading stage, (23),

$$
w_t^B \geq c_t^B + \rho_t^{B,h} s_t^B + (p_t^{B,h} - \rho_t^{B,h}) h_t^B + p_t^{B,n} n_t^B + d_t^{B,nr} - \bar{m}_t^B,
$$

and the budget constraint at the refinancing stage (15). The definition of the total value of the mortgage portfolio $\bar{m}_t^B$ was given in (21).

7.9.1 Borrower's Problem Characterization

In this section I derive a couple of results that prove substantial for the borrower's problem to become tractable, and later on show that the borrower's problem can be solved as a representative agent problem[112].

Proposition 1 The household's value function (75) can be written as

$$
V^B(w_t^B, \mathcal{Z}_t) = v^B(\mathcal{Z}_t) \cdot \frac{(w_t^B)^{1-\gamma}}{1-\gamma}
$$

where $v^B(\mathcal{Z}_t)$ is a function that will only depend on aggregate state variables.

Proof:

[112]It is worth mentioning that some of the derivations in this section are similar to those found in Diamond and Landvoigt (2021)

Step 1: Rewriting the borrower's problem In this section I rewrite the borrower's problem in order to make it tractable. Define the savings of an individual household as Σ_t^B,

$$\Sigma_t^B = w_t^B - c_t^B - \rho_t^{B,h} s_t^B \implies c_t^B + \rho_t^{B,h} s_t^B = w_t^B - \Sigma_t^B \tag{76}$$

Using the budget constraint at the trading stage (23) we can write,

$$\Sigma_t^B = (p_t^{B,h} - \rho_t^{B,h}) h_t^B + p^{B,n} n_t^B + d_t^{B,nr} - \bar{m}_t^B \tag{77}$$

From now on, equations (76) and (77) will represent the budget constraint at the trading stage. Next for each asset define the quantity per unit of savings as, $\hat{x}_t = \frac{x_t}{\Sigma_t^B}$. The newly defined assets are: housing $\left(\hat{h}_t^B = \frac{h_t^B}{\Sigma_t^B}\right)$; borrower-specific asset $\left(\hat{n}_t^B = \frac{n_t^B}{\Sigma_t^B}\right)$; deposits, for both $l \in \{r, nr\}$ $\left(\hat{d}_t^{B,l} = \frac{d_t^{B,l}}{\Sigma_t^B}\right)$; mortgage principal outstanding asset, for all combinations of $l \in \{r, nr\}$ and $j \in \{frm, arm\}$ $\left(\hat{m}_t^{l,j} = \frac{m_t^{l,j}}{\Sigma_t^B}\right)$; FRMs interest payment , for both $l \in \{r, nr\}$ $\left(\hat{m}_t^{I,frm} = \frac{m_t^{I,frm}}{\Sigma_t^B}\right)$; ARMs spred-interest payment , for both $l \in \{r, nr\}$ $\left(\hat{m}_t^{S,arm} = \frac{m_t^{S,arm}}{\Sigma_t^B}\right)$.

Wealth Levels Using these, It will now redefine the post-default wealth levels introduced in section 3.5.1. The non-housing wealth (7) per unit of savings is

$$\hat{w}_{t+1}^{B,nh,l} = \frac{w_{t+1}^{B,nh,l}}{\Sigma_t^B} = (y_{t+1}^{B,n} + p_{t+1}^{B,n}) \cdot \hat{n}_t^B + r_{t+1}^d \cdot \hat{d}_t^{B,l}. \tag{78}$$

For the non-defaulters (8) we have the following expression for $l \in \{r, nr\}$,

$$\begin{aligned}\hat{w}_{t+1}^{B,nd,l}(\epsilon_{t+1}) = \frac{w_{t+1}^{B,nd,l}(\epsilon_{t+1})}{\Sigma_t^B} &= \hat{w}_{t+1}^{B,nh,l} - \hat{T}_{t+1}^B + \left(1 - \delta^h\right) p_{t+1}^{B,h} \cdot \epsilon_{t+1} \cdot \hat{h}_t^B \\ &- \sum_{j=\{arm,frm\}} \left(\hat{\text{payment}}_{t+1}^{l,j} \cdot (1 + \delta \cdot p_{t+1}^{B,I,j}) + (1 - \delta^j) \cdot \hat{m}_t^{l,j} + \delta^j \cdot p_{t+1}^{B,m,j} \cdot \hat{m}_t^{l,j}\right).\end{aligned} \tag{79}$$

where the tax function per unit of savings is, $\hat{T}^B_{t+1} = \frac{T^B_{t+1}}{\Sigma_t} = \tau^B_{t+1} \cdot (\nu \cdot Y_{t+1}) \cdot \hat{n}^B_t$. Furthermore, the mortgage payment functions (9) and (10) can be described as,

$$\hat{\text{payment}}^{l,frm}_{t+1} = \begin{cases} \hat{m}^{I,frm}_t & \text{if } l = nr \\ \underbrace{\iota^{frm}\left(\hat{\alpha}^{B,r}_t, \mathcal{Z}_t\right)}_{\text{Pricing function set at } t} \;\; \hat{m}^{r,frm}_t & \text{if } l = r \end{cases} \tag{80}$$

$$\hat{\text{payment}}^{l,arm}_{t+1} = \begin{cases} \hat{m}^{S,arm}_t + \theta^m \cdot i_{t+1} \cdot \hat{m}^{nr,arm}_t & \text{if } l = nr \\ \Big(\underbrace{spread^{arm}\left(\hat{\alpha}^{B,r}_t, \mathcal{Z}_t\right)}_{\text{Pricing function set at } t} + \theta^m \cdot i_{t+1}\Big) \cdot \hat{m}^{r,arm}_t & \text{if } l = r \end{cases} \tag{81}$$

Notice that the pricing functions have as input the entire portfolio vector chosen at the refinancing stage, $\hat{\alpha}^{B,r}_t = \{\hat{h}^B_t, \hat{n}^B_t, \hat{d}^{B,\bar{r}}_t, \hat{m}^{\bar{r},arm}_t, \hat{m}^{\bar{r},frm}_t\}$. A key aspect of equation (79) is that for it to hold I need the pricing functions in equations (80) and (81) to be homogeneous of degree zero in the household's vector choice $\hat{\alpha}^{B,r}_t$,

$$\iota^{frm}\left(\hat{\alpha}^{B,r}_t, \mathcal{Z}_t\right) = \iota^{frm}\left(\alpha^{B,r}_t, \mathcal{Z}_t\right)$$
$$spread^{arm}\left(\hat{\alpha}^{B,r}_t, \mathcal{Z}_t\right) = spread^{arm}\left(\alpha^{B,r}_t, \mathcal{Z}_t\right)$$

Turns out this is in fact true, but the proof of this specific property can be found in this Appendix in section 7.20 , where I discuss the financial intermediary's problem in detail.

If the agent defaults on mortgage type $j \in \{arm, frm\}$, then the wealth available (11) per unit of savings can be written as,

$$\begin{aligned} \hat{w}^{B,(d,j),l}_{t+1}(\epsilon_{t+1}) = \frac{w^{B,(d,j),l}_{t+1}(\epsilon_{t+1})}{\Sigma^B_t} &= (1-\lambda^j)\hat{w}^{B,nh,l}_{t+1} - \hat{T}^B_{t+1} + \left(1-\delta^h\right) p^{B,h}_{t+1} \cdot \epsilon_t \cdot \hat{h}^B_t \cdot \hat{\chi}^{l,k}_t \\ &- \left(\hat{\text{payment}}^{l,k}_{t+1} \cdot (1 + \delta \cdot p^{B,I,\kappa}_{t+1}) + (1-\delta^k) \cdot \hat{m}^{l,k}_t + \delta^k \cdot p^{B,m,\kappa}_{t+1} \cdot \hat{m}^{l,k}_t\right), \end{aligned} \tag{82}$$

where I am still using the homogeneity of degree zero of the pricing functions property discussed previously.

Furthermore, notice that

$$\chi_t^{l,k} = \frac{m_t^{l,k}}{m_t^{l,k} + m_t^{l,k}} = \frac{\hat{m}_t^{l,k}}{\hat{m}_t^{l,k} + \hat{m}_t^{l,k}} = \hat{\chi}_t^{l,k}$$

Finally, if the agent defaults on all its mortgage balance, the wealth (13) per unit available is,

$$\hat{w}_{t+1}^{B,d,l} = \frac{w_{t+1}^{B,d,l}}{\Sigma_t^B} = (1 - \lambda^{arm} - \lambda^{frm}) \cdot \hat{w}_{t+1}^{B,nh,l} - \hat{T}_{t+1}^B, \tag{83}$$

Budget Constraints The definition of the savings Σ_t^B at the trading stage (77) can be written as,

$$1 = (p_t^{B,h} - \rho_t^{B,h})\hat{h}_t^B + p^{B,n}\hat{n}_t^B + \hat{d}_t^{B,nr} - \hat{\bar{m}}_t^B \tag{84}$$

where $\hat{\bar{m}}_t^B = p_t^{B,m,frm}\hat{m}_t^{nr,frm} + p_t^{B,m,arm}\hat{m}_t^{nr,arm} + p_t^{B,I,frm}\hat{m}_t^{I,frm} + p_t^{B,I,arm}\hat{m}_t^{S,arm}$.

Finally, the budget constraint at the refinancing stage (15) can be written as,

$$\hat{d}_t^{B,nr} - \sum_{j=\{arm,frm\}} \hat{m}_t^{nr,j} = \hat{d}_t^{B,\bar{r}} - \sum_{j=\{arm,frm\}} \left[\hat{m}_t^{r,j} - \hat{m}_t^{r,j} \cdot C\left(\frac{\hat{m}_t^{r,j}}{\hat{h}_t^B}\right) \right] \tag{85}$$

Step 2: Split the Borrower's Problem The next step is to split the borrower's problem in order to define an auxiliary portfolio choice problem that does not depend on the wealth of the borrower. This is a first step towards the aggregation result. Notice that choosing the optimal consumption (c_t^B) and housing service (s_t^B) from problem (75) subject to the (recasted) budget constraint[113] (76) is simple, we can compute those using the typical Cobb-Douglas solution,

$$\begin{aligned} c_t^B &= \theta^B \cdot \left(w_t^B - \Sigma_t^B\right) \\ s_t^B &= \frac{(1-\theta^B)}{\rho_t^{B,h}} \cdot \left(w_t^B - \Sigma_t^B\right) \end{aligned} \tag{86}$$

We can plug these solution in the Bellman equation (75) to obtain,

[113] These variables do not appear in the refinancing stage budget constraint.

$$
\begin{aligned}
V^B(w_t^B, \mathcal{Z}_t) = &\max_{\left\{\Sigma_t^B, \alpha_t^{B,\overline{nr}}, \alpha_t^{B,\overline{r}}\right\}} \frac{C^B(\mathcal{Z}_t)}{1-\gamma}\left(w_t^B - \Sigma_t^B\right)^{1-\gamma} + \psi^B \cdot \frac{(d_t^{B,nr})^{1-\gamma}}{1-\gamma} \\
&+ \sum_{l=\{r,nr\}} \beta^B \cdot q^l \cdot E_t\Bigg[\max\Bigg(V^B\left(w_{t+1}^{B,nd,l}, \mathcal{Z}_{t+1}\right), V^B\left(w_{t+1}^{B,(d,arm),l}, \mathcal{Z}_{t+1}\right), \\
&\qquad V^B\left(w_{t+1}^{B,(d,frm),l}, \mathcal{Z}_{t+1}\right), V^B\left(w_{t+1}^{B,d,l}, \mathcal{Z}_{t+1}\right)\Bigg)\Bigg]
\end{aligned} \tag{87}
$$

where the constant term $C^B(\mathcal{Z}_t) = \left(\kappa^\kappa + \left(\frac{1-\kappa}{\rho_t^{B,h}}\right)^{1-\kappa}\right)^{1-\gamma}$. Notice that now I are optimizing with respect to the savings share Σ_t^B. This new problem still depends on the wealth level w_t^B, therefore now I define the *portfolio choice problem per dollar of savings*, $W^B(\mathcal{Z}_t)$. For this purpose, I will state the properties that I am using to define this new problem,

1. First, we will conjecture and then verify that the value function has the form

$$
V^B(w_t^B, \mathcal{Z}_t) = v^B(\mathcal{Z}_t)\frac{(w_t^B)^{1-\gamma}}{1-\gamma}
$$

2. The budget set need to satisfy the following property: the choice vector $\left\{c_t^B, s_t^B, \alpha_t^{B,nr}, \alpha_t^{B,r}\right\}$ satisfies the budget set with wealth level $w_t^B \iff$
for some constant $k > 0$ the choice vector $\left\{k \cdot c_t^B, k \cdot s_t^B, k \cdot \alpha_t^{B,nr}, k \cdot \alpha_t^{B,r}\right\}$ satisfies the budget set with wealth level $k \cdot w_t^B$. We already used this property which allowed us to write the budget set as shown in equations (84) and (85)[114] in Step 1.

3. All the realizations of household's post-default wealth levels in period $t+1$ are linear functions of the household's portfolio choice vector $\left\{\alpha_t^{B,nr}, \alpha_t^{B,r}\right\}$. We also already used this property which allowed to rewrite the wealth levels (79), (82), and (83) in Step 1.

This new problem can be written as,

$$
\begin{aligned}
W^B(\mathcal{Z}_t) = &\max_{\left\{\alpha_t^{B,nr}, \alpha_t^{B,r}\right\}} \quad \psi^B \cdot \frac{(\hat{d}_t^{B,nr})^{1-\gamma}}{1-\gamma} + \sum_{l=\{r,nr\}} \beta^B \cdot q^l \cdot E_t\Bigg[\frac{v^B(\mathcal{Z}_{t+1})}{1-\gamma} \cdot \\
&\max\left(\left(\hat{w}_{t+1}^{B,nd,l}\right)^{1-\gamma}, \left(\hat{w}_{t+1}^{B,(d,arm),l}\right)^{1-\gamma}, \left(\hat{w}_{t+1}^{B,(d,frm),l}\right)^{1-\gamma}, \left(\hat{w}_{t+1}^{B,d,l}\right)^{1-\gamma}\right)\Bigg]
\end{aligned} \tag{88}
$$

[114] In that case, $k = \Sigma_t^B$

subject to to the budget constraints (84) and (85). (88) shows that the objective function at time t is homogeneous of degree $1-\gamma$ in the household's choice vector. Furthermore, the original value function (87) can be defined as,

$$V^B(w_t^B, \mathcal{Z}_t) = \max_{\Sigma_t^B} \frac{C^B(\mathcal{Z}_t)}{1-\gamma} \left(w_t^B - \Sigma_t^B\right)^{1-\gamma} + \left(\Sigma_t^B\right)^{1-\gamma} W^B(\mathcal{Z}_t) \tag{89}$$

Notice that the problem (88) is independent of individual wealth, all borrowers choose the same portfolio and savings shares, irrespective of their level of wealth at the beginning of this stage.

Step 3: Find the optimal value of Σ_t^B The final step is to solve the simple problem (89), and find the optimal savings value for Σ_t^B,

$$\Sigma_t^B = \frac{\left[(1-\gamma)\cdot W^B(\mathcal{Z}_t)\right]^{\frac{1}{\gamma}}}{C^B(\mathcal{Z}_t)^{\frac{1}{\gamma}} + [(1-\gamma)\cdot W^B(\mathcal{Z}_t)]^{\frac{1}{\gamma}}} \cdot w_t^B \tag{90}$$

Where the fraction of wealth saved is defined as,

$$\sigma_t^B = \frac{\left[(1-\gamma)\cdot W^B(\mathcal{Z}_t)\right]^{\frac{1}{\gamma}}}{C^B(\mathcal{Z}_t)^{\frac{1}{\gamma}} + [(1-\gamma)\cdot W^B(\mathcal{Z}_t)]^{\frac{1}{\gamma}}}$$

Plug-in the Σ_t^B solution into the Bellman equation.

$$V^B(w_t^B, \mathcal{Z}_t) = \frac{(w_t^B)^{1-\gamma}}{1-\gamma}\left[C^B(\mathcal{Z}_t)(1-\sigma_t^B)^{1-\gamma} + (1-\gamma)\cdot W^B(\mathcal{Z}_t)(\sigma_t^B)^{1-\gamma}\right]$$

which verifies the conjecture of the functional form of $V^B(w_t^B, \mathcal{Z}_t)$. We just need to set

$$v^B(\mathcal{Z}_t) = C^B(\mathcal{Z}_t)(1-\sigma_t^B)^{1-\gamma} + (1-\gamma)\cdot W^B(\mathcal{Z}_t)(\sigma_t^B)^{1-\gamma}$$

which is a recursion on $v^B(\mathcal{Z}_t)$ since the value function $W^B(\mathcal{Z}_t)$ depends on the expected values of $v^B(\mathcal{Z}_{t+1})$.
□

Proposition 2 If some household $B1$ enters the trading stage with post-default wealth w_t^{B1} such that its optimal choice vector is $\{c_t^{B1}, s_t^{B1}, \alpha_t^{B1,nr}, \alpha_t^{B1,r}\}$,then for any other household $B2$ who enters the trading stage with post-default wealth w_t^{B2} such that $w_t^{B2} = k\cdot w_t^{B1}$, then its optimal choice vector will exactly equal

$\{k \cdot c_t^{B1}, k \cdot s_t^{B1}, k \cdot \alpha_t^{B1,nr}, k \cdot \alpha_t^{B1,r}\}$.

Proof: Let $w_t^{B2} = k \cdot w_t^{B1}$. By definition the *portfolio choice problem per dollar of savings*, $W^B(\mathcal{Z}_t)$ of both agents is identical (since we are scaling it by the amount of savings Σ_t^B). However,

$$w_t^{B2} - \Sigma_t^{B2} = k \cdot (w_t^{B1} - \Sigma_t^{B1}),$$

where the equality follows the definition of the optimal savings Σ_t^B derived in equation (90) in Proposition 1. That implies that $c_t^{B2} = k \cdot c_t^{B1}$, using equations (86). Lastly, $\Sigma_t^{B2} = k \cdot \Sigma_t^{B1}$, which implies $\alpha_t^{B2,nr} = k \cdot \alpha_t^{B1,nr}$ and $\alpha_t^{B2,r} = k \cdot \alpha_t^{B1,r}$ given the scaling properties of $W^B(\mathcal{Z}_t)$. □

7.10 Intuition Default Thresholds

I will provide more intuition abut the default threshold values. In particular, consider a log-normal cumulative distribution function $F : \mathbb{E} \mapsto [0, 1]$, with a respective probability distribution function $f : \mathbb{E} \mapsto \mathbb{R}$, where $\mathbb{E}$ is the domain of idiosyncratic shock $\epsilon_{t+1} \in \{0, \infty\}$. For any state z_{t+1} at the default stage, and standing in period t at the trading stage, the individual borrower needs to take care of two cases. case i) states where $\hat{\epsilon}_{t+1}^{l,arm} < \hat{\epsilon}_{t+1}^{l,frm}$, or case ii) states where $\hat{\epsilon}_{t+1}^{l,arm} > \hat{\epsilon}_{t+1}^{l,frm}$, for each $l \in \{r, nr\}$. In figure 29, I show a graphic representation of case i).

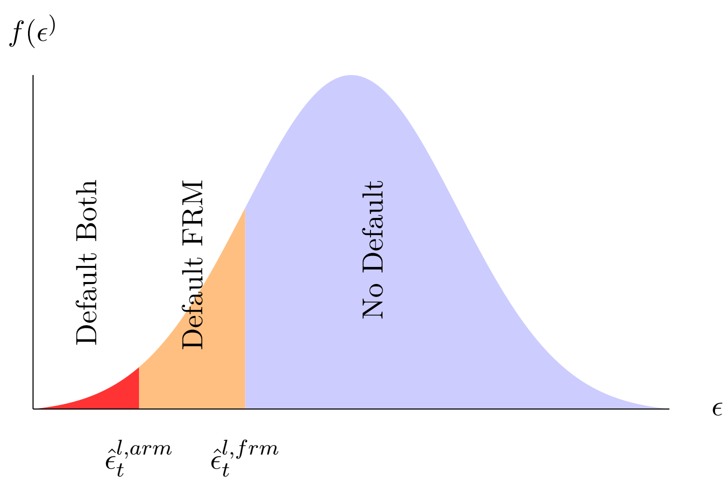


Figure 29: Representation of the default thresholds

If we are in case i), all values of ϵ below $\hat{\epsilon}_t^{l,arm}$ represent all the borrowers that default on their adjustable rate mortgages (ARMs), hence $F(\hat{\epsilon}_t^{l,arm})$ equals the default rate on ARMs. All values of ϵ below $\hat{\epsilon}_t^{l,frm}$ represent all the borrowers that default on their fixed rate mortgages (FRMs), hence $F(\hat{\epsilon}_t^{l,frm})$ equals the default rate on FRMs. In this specific case we have,

1. $F(\hat{\epsilon}_t^{l,arm})$ also equals the default rate of those borrowers that default on both mortgages;

2. $F(\hat{\epsilon}_t^{l,frm}) - F(\hat{\epsilon}_t^{l,arm})$ represents the default rate of those agents that *only* default on FRMs;

3. $1 - F(\hat{\epsilon}_t^{l,frm})$ represents the repayment rate.

Therefore, when $\hat{\epsilon}_t^{l,arm} < \hat{\epsilon}_t^{l,frm}$ we do not have borrowers defaulting on *only* on ARMs. A similar logic follows from case ii) when $\hat{\epsilon}_t^{l,arm} > \hat{\epsilon}_t^{l,frm}$.

7.11 Derivation of Default Thresholds

In this Appendix I will compute the threshold values $\hat{\epsilon}_t^{l,arm}$ and $\hat{\epsilon}_t^{l,frm}$. I will use the post-default wealth definitions (8), (11), and (13)[115]. As in the main text, I will split the derivations in two cases since it helps the model's intuition. Case i) $\hat{\epsilon}_t^{l,arm} < \hat{\epsilon}_t^{l,frm}$ or Case ii) $\hat{\epsilon}_t^{l,frm} < \hat{\epsilon}_t^{l,arm}$. I can derive the expressions for the thresholds using any of the two cases, and obtain the same results. First, I do for case i) and later i do it for case ii) just for completeness. Notice that these computations extend for both $l \in \{r, nr\}$

Figure 29 shows the logic under case i), to compute the threshold value for the ARM, the two relevant wealth values we need to equalize are *the only-FRM default*, $w_{t+1}^{B,(d,frm),l}$, and *the total default*, $w_{t+1}^{B,d,l}$. The equality between these two wealth levels only occur at $\epsilon_t = \hat{\epsilon}_t^{l,arm}$,

$$w_{t+1}^{B,(d,frm),l}(\hat{\epsilon}_{t+1}^{l,arm}) = w_{t+1}^{B,d,l} \tag{91}$$

Substituting the wealth expressions in (91),

$$\begin{aligned}
&(1-\lambda^{frm}) \cdot w_{t+1}^{B,nh,l} - T_{t+1}^{B} + \left(1-\delta^h\right) p_{t+1}^{B,h} \cdot \hat{\epsilon}_{t+1}^{l,arm} \cdot h_t^B \cdot \chi_t^{l,arm} - \\
&\left(\frac{\text{payment}_{t+1}^{l,arm}}{m_t^{l,arm}} \cdot (1+\delta \cdot p_{t+1}^{B,I,arm}) + (1-\delta) + \delta \cdot p_{t+1}^{B,m,arm}\right) m_t^{l,arm} \\
&= (1-\lambda^{B,arm} - \lambda^{B,frm}) w_{t+1}^{B,nh,l}
\end{aligned} \tag{92}$$

[115] Alternatively, I could use (79), (82), and (83). Since the default thresholds resemble loan to value ratios (i.e. divisions of the assets that define the portfolio choice), it does not make a difference.

Alternatively,

$$
\begin{aligned}
&\hat{\epsilon}^{l,arm}_{t+1} \cdot \underbrace{\left(1-\delta^{h}\right) p^{B,h}_{t+1} \cdot h^{B}_{t} \cdot \chi^{l,arm}_{t}}_{\text{Housing value funded with an ARM}} \\
&= \underbrace{\left(\frac{\text{payment}^{l,arm}_{t+1}}{m^{l,arm}_{t}} \cdot (1+\delta \cdot p^{B,I,arm}_{t+1}) + (1-\delta) + \delta \cdot p^{B,m,arm}_{t+1} \right) m^{l,arm}_{t}}_{\text{Mortgage Market Value = Cost of not defaulting on the ARM}} - \underbrace{\lambda^{B,arm} w^{B,nh,l}_{t+1}}_{\text{Cost of defaulting on the ARM}}
\end{aligned} \tag{93}
$$

Expression (93) explains how the model endogenously generates default. If the market value of the ARMs debt is large, this will increase $\hat{\epsilon}^{l,arm}_{t+1}$ which will force a large fraction of borrowers to default, this makes sense since the debt burden is higher. Notice that the structure of the mortgage payments are an important part of this term. On the other hand, if the housing value funded with ARMs is large or the non-housing wealth is large (which is partially lost after default), the value of $\hat{\epsilon}^{l,arm}_{t+1}$ will decrease which will make a smaller fraction of borrowers to default; in other words, default is costly since defaulting decreases the value of the wealth entering into the trading stage which is used for non-durable consumption and housing service purchases. To compute the threshold value for the FRM, the two relevant wealth values (see figure 29) we need to equalize are *the only-FRM default*, $w^{B,(d,frm),l}_{t+1}$, and *no default*, $w^{B,nd,l}_{t+1}$. The equality between these two wealth levels only occur at $\epsilon_t = \hat{\epsilon}^{l,frm}_{t}$,

$$
w^{B,(d,frm),l}_{t+1}(\hat{\epsilon}^{l,frm}_{t+1}) = w^{B,nd,l}_{t+1}(\hat{\epsilon}^{l,frm}_{t+1}) \tag{94}
$$

Substituting the wealth expressions in (94),

$$
\begin{aligned}
&(1-\lambda^{frm}) \cdot w^{B,nh,l}_{t+1} - T^{B}_{t+1} + \left(1-\delta^{h}\right) p^{B,h}_{t+1} \cdot \hat{\epsilon}^{l,frm}_{t+1} \cdot h^{B}_{t} \cdot \chi^{l,arm}_{t} - \\
&\left(\frac{\text{payment}^{l,arm}_{t+1}}{m^{l,arm}_{t}} \cdot (1+\delta \cdot p^{B,I,arm}_{t+1}) + (1-\delta) + \delta \cdot p^{B,m,arm}_{t+1} \right) m^{l,arm}_{t} \\
&= w^{B,nh,l}_{t+1} - T^{B}_{t+1} + \left(1-\delta^{h}\right) p^{B,h}_{t+1} \cdot \hat{\epsilon}^{l,frm}_{t+1} \cdot h^{B}_{t} - \\
&\sum_{j=\{arm,frm\}} \left(\frac{\text{payment}^{l,j}_{t+1}}{m^{l,j}_{t}} \cdot (1+\delta \cdot p^{B,I,j}_{t+1}) + (1-\delta^{j}) + \delta^{j} \cdot p^{B,m,j}_{t+1} \right) m^{l,j}_{t}.
\end{aligned} \tag{95}
$$

Alternatively,

$$\hat{\epsilon}_{t+1}^{l,frm} \cdot \underbrace{\left(1-\delta^{h}\right) p_{t+1}^{B,h} \cdot h_{t}^{B} \cdot \chi_{t}^{l,frm}}_{\text{Housing value funded with an FRM}}$$

$$= \underbrace{\left(\frac{\text{payment}_{t+1}^{l,frm}}{m_{t}^{l,frm}} \cdot (1+\delta \cdot p_{t+1}^{B,I,frm}) + (1-\delta) + \delta \cdot p_{t+1}^{B,m,frm}\right) m_{t}^{l,frm}}_{\text{Mortgage Market Value = Cost of not defaulting on the FRM}} - \underbrace{\lambda^{B,frm} w_{t+1}^{B,nh,l}}_{\text{Cost of defaulting on the FRM}} \tag{96}$$

where we used the relation $\chi_t^{l,arm} + \chi_t^{l,frm} = 1$. (96) closely resembles (93) and it explains how the model endogenously generates default on FRMs. Again, the borrower should compare the fixed-rate debt burden relative to its housing wealth loss after the FRMs and the size of the housing funded with this type of debt. Finally, it is interesting to ask for which states z_{t+1} is the borrower in case i) $\hat{\epsilon}_t^{l,arm} < \hat{\epsilon}_t^{l,frm}$, for that to be true we require,

$$\frac{mmv_{t+1}^{l,arm} - \lambda^{arm} w_{t}^{B,nh,l}}{(1-\delta^{h})\, p_{t+1}^{B,h} \cdot h_{t}^{B} \cdot \chi_{t}^{arm,l}} < \frac{mmv_{t+1}^{l,frm} - \lambda^{frm} w_{t}^{B,nh,l}}{(1-\delta^{h})\, p_{t+1}^{B,h} \cdot h_{t}^{B} \cdot \chi_{t}^{frm,l}}$$
$$\Rightarrow \frac{mmv_{t+1}^{l,arm} - \lambda^{arm} w_{t}^{B,nh,l}}{\chi_{t}^{arm,l}} < \frac{mmv_{t+1}^{l,frm} - \lambda^{frm} w_{t}^{B,nh,l}}{\chi_{t}^{frm,l}} \tag{97}$$

For convenience, define the mortgage market value, for $l \in \{r, nr\}$ and $j \in \{frm, arm\}$, as

$$mmv_{t+1}^{l,j} = \left(\frac{\text{payment}_{t+1}^{l,j}}{m_{t}^{l,j}} \cdot (1+\delta \cdot p_{t+1}^{B,I,j}) + (1-\delta^{j}) + \delta^{j} \cdot p_{t+1}^{B,m,j}\right) m_{t}^{l,j}.$$

Computing Default Thresholds under case ii) For completeness, I compute the default thresholds $\hat{\epsilon}_t^{l,arm}$ and $\hat{\epsilon}_t^{l,frm}$ but under case ii) $\hat{\epsilon}_t^{l,frm} < \hat{\epsilon}_t^{l,arm}$. For this case, since $\hat{\epsilon}_t^{l,frm}$ is smaller we do not have the *only-FRM* default case.

To compute the threshold value for the ARM, the two relevant wealth values we need to equalize are *the only-ARM default*, $w_{t+1}^{B,(d,arm),l}$, and *the no default*, $w_{t+1}^{B,nd,l}$. The equality between these two wealth levels only occur at $\epsilon_t = \hat{\epsilon}_t^{l,arm}$,

$$w_{t+1}^{B,(d,arm),l}(\hat{\epsilon}_{t+1}^{l,arm}) = w_{t+1}^{B,nd,l}(\hat{\epsilon}_{t+1}^{l,arm}) \tag{98}$$

Substituting the wealth expressions in (98),

$$
\begin{aligned}
&(1-\lambda^{arm})\cdot w_{t+1}^{B,nh,l} - T_{t+1}^{B} + \left(1-\delta^{h}\right) p_{t+1}^{B,h}\cdot \hat{\epsilon}_{t+1}^{l,arm}\cdot h_t^B \cdot \chi_t^{l,frm} - \\
&\left(\frac{\text{payment}_{t+1}^{l,frm}}{m_t^{l,frm}}\cdot(1+\delta\cdot p_{t+1}^{B,I,frm}) + (1-\delta) + \delta\cdot p_{t+1}^{B,m,frm}\right) m_t^{l,frm} \\
&= w_{t+1}^{B,nh,l} - T_{t+1}^{B} + \left(1-\delta^{h}\right) p_{t+1}^{B,h}\cdot \hat{\epsilon}_{t+1}^{l,arm}\cdot h_t^B - \\
&\sum_{j=\{arm,frm\}} \left(\frac{\text{payment}_{t+1}^{l,j}}{m_t^{l,j}}\cdot(1+\delta\cdot p_{t+1}^{B,I,j}) + (1-\delta^{j}) + \delta^{j}\cdot p_{t+1}^{B,m,j}\right) m_t^{l,j}.
\end{aligned} \tag{99}
$$

Alternatively,

$$
\begin{aligned}
&\hat{\epsilon}_{t+1}^{l,arm}\cdot \underbrace{\left(1-\delta^{h}\right) p_{t+1}^{B,h}\cdot h_t^B\cdot \chi_t^{l,arm}}_{\text{Housing value funded with an ARM}} \\
&= \underbrace{\left(\frac{\text{payment}_{t+1}^{l,arm}}{m_t^{l,arm}}\cdot(1+\delta\cdot p_{t+1}^{B,I,arm}) + (1-\delta) + \delta\cdot p_{t+1}^{B,m,arm}\right) m_t^{l,arm}}_{\text{Mortgage Market Value = Cost of not defaulting on the ARM}} - \underbrace{\lambda^{arm} w_{t+1}^{B,nh,l}}_{\text{Cost of defaulting on the ARM}}
\end{aligned} \tag{100}
$$

This is the same expression we got on equation (93). Finally, to compute the threshold value for the FRM, the two relevant wealth values we need to equalize are *the only-FRM default*, $w_{t+1}^{B,(d,arm),l}$, and *total default*, $w_{t+1}^{B,d,l}$. The equality between these two wealth levels only occur at $\epsilon_t = \hat{\epsilon}_t^{l,frm}$,

$$
w_{t+1}^{B,(d,arm),l}(\hat{\epsilon}_{t+1}^{l,frm}) = w_{t+1}^{B,d,l} \tag{101}
$$

Substituting the wealth expressions in (101),

$$
\begin{aligned}
&(1-\lambda^{arm})\cdot w_{t+1}^{B,nh,l} - T_{t+1}^{B} + \left(1-\delta^{h}\right) p_{t+1}^{B,h}\cdot \hat{\epsilon}_{t+1}^{l,frm}\cdot h_t^B \cdot \chi_t^{l,frm} - \\
&\left(\frac{\text{payment}_{t+1}^{l,frm}}{m_t^{l,frm}}\cdot(1+\delta\cdot p_{t+1}^{B,I,frm}) + (1-\delta) + \delta\cdot p_{t+1}^{B,m,frm}\right) m_t^{l,frm} \\
&= (1-\lambda^{B,arm} - \lambda^{B,frm}) w_{t+1}^{B,nh,l}
\end{aligned} \tag{102}
$$

Alternatively,

$$
\hat{\epsilon}_{t+1}^{l,frm} \cdot \underbrace{\left(1-\delta^h\right) p_{t+1}^{B,h} \cdot h_t^B \cdot \chi_t^{l,frm}}_{\text{Housing value funded with an FRM}}
$$

$$
= \underbrace{\left(\frac{\text{payment}_{t+1}^{l,frm}}{m_t^{l,frm}} \cdot (1 + \delta \cdot p_{t+1}^{B,I,frm}) + (1-\delta) + \delta \cdot p_{t+1}^{B,m,frm} \right) m_t^{l,frm}}_{\text{Mortgage Market Value = Cost of not defaulting on the FRM}} - \underbrace{\lambda^{B,frm} w_{t+1}^{B,nh,l}}_{\text{Cost of defaulting on the FRM}} \tag{103}
$$

This is the same expression we got on equation (96).

7.12 Borrower's First Order Conditions

Using the results derived in 3.5.4, the borrower's problem (22) can be written as

$$
V^B(w_t^B, \mathcal{Z}_t) = \max_{\{c_t^B, s_t^B, \alpha_t^{B,nr}, \alpha_t^{B,r}\}} \frac{\left(\left(c_t^B\right)^{\theta^B} \left(s_t^B\right)^{1-\theta^B} \right)^{1-\gamma}}{1-\gamma} + \frac{\psi^B \cdot (d_t^{B,\overline{nr}})^{1-\gamma}}{1-\gamma}
$$

$$
+ \sum_{l=\{r,nr\}} \beta^B \cdot q^l E_t \left[\frac{v^B(\mathcal{Z}_{t+1})}{1-\gamma} \cdot \mathbb{1}_{\{\hat{\epsilon}_t^{l,arm} < \hat{\epsilon}_t^{l,frm}\}} \max \left(\left(w_{t+1}^{B,nd,l}\right)^{1-\gamma}, \left(w_{t+1}^{B,\bar{d},l}\right)^{1-\gamma}, \left(w_{t+1}^{B,(d,frm),l}\right)^{1-\gamma} \right) \right.
$$

$$
\left. + \frac{v^B(\mathcal{Z}_{t+1})}{1-\gamma} \cdot \mathbb{1}_{\{\hat{\epsilon}_t^{l,arm} > \hat{\epsilon}_t^{l,frm}\}} \max \left(\left(w_{t+1}^{B,\overline{nd},l}\right)^{1-\gamma}, \left(w_{t+1}^{B,\bar{d},l}\right)^{1-\gamma}, \left(w_{t+1}^{B,(d,arm),l}\right)^{1-\gamma} \right) \right] \tag{104}
$$

subject to the budget constraint at the trading stage (23) and refinancing stage (15), and the pricing functions (16). This is the final version of the borrower's Bellman equation I solve for.

In this section, I derive the first order conditions of the borrower's problem as presented in (104). Since the solution to the optimization problem of households scales in individual wealth (Proposition 1 & 2), we can construct stochastic discount factors for a representative borrower. Wealth in period $t+1$ per unit of wealth at period t can be written as follows, for $l \in \{r, nr\}$ and $k \in \{d, nd, (d,j)\}$ for $j \in frm, arm$, using (90)

$$
\frac{w_{t+1}^{B,k,j}}{w_t^B} = \frac{\hat{w}_{t+1}^{B,nd,j} \cdot \Sigma_t^B}{w_t^B} = \frac{\hat{w}_{t+1}^{B,nd,j} \cdot \sigma_t^B \cdot w_t^B}{w_t^B} = \hat{w}_{t+1}^{B,nd,j} \cdot \sigma_t^B
$$

where we used the definitions of some variables in section 7.9 .

7.12.1 Preamble for the FOCs

First, by the envelope condition, the multiplier on the budget constraints (23) is

$$\mu_t^{B,nr} = \frac{v^B(\mathcal{Z}_t)}{\sigma^B(\mathcal{Z}_t)^{-\gamma}},$$

As explained before, we can construct stochastic discount factors ($SDFs$) for a representative borrower, we do this here and later use these definitions in the first order conditions expressions in (7.12.2). These $SDFs$ will apply to both refinancing status $l \in \{r, nr\}$. For the case i) $\hat{\epsilon}_{t+1}^{j,arm} < \hat{\epsilon}_{t+1}^{j,frm}$ described in section (3.5.4), where we observe *only-FRM default*,

$$\mathcal{S}_{arm<frm}^{B,nd,j} = \beta^B \cdot q^j \cdot \mathbb{1}_{\{\hat{\epsilon}_{t+1}^{j,arm} < \hat{\epsilon}_{t+1}^{j,frm}\}} \cdot \frac{\sigma^B(\mathcal{Z}_t)^{-\gamma} \cdot v^B(\mathcal{Z}_{t+1})}{v^B(\mathcal{Z}_t)} \left[\int_{\hat{\epsilon}_{t+1}^{j,frm}}^{\infty} \cdot \left(\hat{w}_{t+1}^{B,nd,j} \right)^{-\gamma} \right] \tag{105}$$

$$\mathcal{S}_{arm<frm}^{B,(d,frm),j} = \beta^B \cdot q^j \cdot \mathbb{1}_{\{\hat{\epsilon}_{t+1}^{j,arm} < \hat{\epsilon}_{t+1}^{j,frm}\}} \cdot \frac{\sigma^B(\mathcal{Z}_t)^{-\gamma} \cdot v^B(\mathcal{Z}_{t+1})}{v^B(\mathcal{Z}_t)} \left[\int_{\hat{\epsilon}_{t+1}^{r,arm}}^{\hat{\epsilon}_{t+1}^{r,frm}} \cdot \left(\hat{w}_{t+1}^{B,(d,frm),j} \right)^{-\gamma} \right] \tag{106}$$

$$\mathcal{S}_{arm<frm}^{B,d,j} = \beta^B \cdot q^j \cdot \mathbb{1}_{\{\hat{\epsilon}_{t+1}^{j,arm} < \hat{\epsilon}_{t+1}^{j,frm}\}} \cdot \frac{\sigma^B(\mathcal{Z}_t)^{-\gamma} \cdot v^B(\mathcal{Z}_{t+1})}{v^B(\mathcal{Z}_t)} \left[\int_{0}^{\hat{\epsilon}_{t+1}^{r,arm}} \cdot \left(\hat{w}_{t+1}^{B,d,j} \right)^{-\gamma} \right] \tag{107}$$

For the case ii) $\hat{\epsilon}_{t+1}^{j,frm} < \hat{\epsilon}_{t+1}^{j,arm}$ described in section (3.5.4), where we observe *only-ARM default*,

$$\mathcal{S}_{frm<arm}^{B,nd,j} = \beta^B \cdot q^j \cdot \mathbb{1}_{\{\hat{\epsilon}_{t+1}^{j,frm} < \hat{\epsilon}_{t+1}^{j,arm}\}} \cdot \frac{\sigma^B(\mathcal{Z}_t)^{-\gamma} \cdot v^B(\mathcal{Z}_{t+1})}{v^B(\mathcal{Z}_t)} \left[\int_{\hat{\epsilon}_{t+1}^{j,arm}}^{\infty} \cdot \left(\hat{w}_{t+1}^{B,nd,j} \right)^{-\gamma} \right] \tag{108}$$

$$\mathcal{S}_{frm<arm}^{B,(d,frm),j} = \beta^B \cdot q^j \cdot \mathbb{1}_{\{\hat{\epsilon}_{t+1}^{j,frm} < \hat{\epsilon}_{t+1}^{j,arm}\}} \cdot \frac{\sigma^B(\mathcal{Z}_t)^{-\gamma} \cdot v^B(\mathcal{Z}_{t+1})}{v^B(\mathcal{Z}_t)} \left[\int_{\hat{\epsilon}_{t+1}^{r,frm}}^{\hat{\epsilon}_{t+1}^{r,arm}} \cdot \left(\hat{w}_{t+1}^{B,(d,frm),j} \right)^{-\gamma} \right] \tag{109}$$

$$\mathcal{S}_{frm<arm}^{B,d,j} = \beta^B \cdot q^j \cdot \mathbb{1}_{\{\hat{\epsilon}_{t+1}^{j,frm} < \hat{\epsilon}_{t+1}^{j,arm}\}} \cdot \frac{\sigma^B(\mathcal{Z}_t)^{-\gamma} \cdot v^B(\mathcal{Z}_{t+1})}{v^B(\mathcal{Z}_t)} \left[\int_{0}^{\hat{\epsilon}_{t+1}^{r,frm}} \cdot \left(\hat{w}_{t+1}^{B,d,j} \right)^{-\gamma} \right] \tag{110}$$

7.12.2 First Order Conditions (FOCs) for the Borrower's Problem

For those borrowers that refinanced their mortgage in period t, the relevant $SDFs$ for the deposits asset are those only for refinancers, notice that deposits are held even under default which affects their future return. Finally, the pricing functions are featured in this FOC since those mortgages were priced in period t.

1. FOC for $\hat{d}_t^{B,r}$:.

$$E_t\Bigg[\mathcal{S}^{B,nd,r}_{t+1,arm<frm}\Bigg(r^d_{t+1}-\hat{m}^{r,frm}_t\left(\frac{\partial\iota^{frm}(\cdot)}{\partial\hat{d}^{B,r}_t}\right)\cdot(1+\delta\cdot p^{B,I,frm}_{t+1})$$

$$-\hat{m}^{r,arm}_t\left(\frac{\partial spread^{arm}(\cdot)}{\partial\hat{d}^{B,r}_t}\right)\cdot(1+\delta\cdot p^{B,I,arm}_{t+1})\Bigg)$$

$$+\mathcal{S}^{B,(d,frm),r}_{t+1,arm<frm}\left((1-\lambda^{frm})\cdot r^d_{t+1}-\hat{m}^{r,arm}_t\left(\frac{\partial spread^{arm}(\cdot)}{\partial\hat{d}^{B,r}_t}\right)\cdot(1+\delta\cdot p^{B,I,arm}_{t+1})\right)$$

$$+\mathcal{S}^{B,d,r}_{t+1,arm<frm}(1-\lambda^{frm}-\lambda^{arm})\cdot r^d_{t+1}$$

$$+\mathcal{S}^{B,nd,r}_{t+1,frm<arm}\Bigg(r^d_{t+1}-\hat{m}^{r,frm}_t\left(\frac{\partial\iota^{I,frm}(\cdot)}{\partial\hat{d}^{B,r}_t}\right)\cdot(1+\delta\cdot p^{B,I,frm}_{t+1})$$

$$-\hat{m}^{r,arm}_t\left(\frac{\partial spread^{arm}(\cdot)}{\partial\hat{d}^{B,r}_t}\right)\cdot(1+\delta\cdot p^{B,I,arm}_{t+1})\Bigg)$$

$$+\mathcal{S}^{B,(d,arm),r}_{t+1,frm<arm}\left((1-\lambda^{arm})\cdot r^d_{t+1}-\hat{m}^{r,frm}_t\left(\frac{\partial\iota^{frm}(\cdot)}{\partial\hat{d}^{B,r}_t}\right)\cdot(1+\delta\cdot p^{B,I,frm}_{t+1})\right)$$

$$+\mathcal{S}^{B,d,r}_{frm<arm}(1-\lambda^{frm}-\lambda^{arm})\cdot r^d_{t+1}\Bigg]=1 \tag{111}$$

For those borrowers that did not refinanced their mortgage in period t, the relevant $SDFs$ for the deposits asset are those only for non-refinancers, notice that deposits are held even under default which affects their future return. No pricing functions are used in this FOC.

2. FOC for $\hat{d}_t^{B,nr}$:.

$$E_t\Bigg[\mathcal{S}^{B,nd,nr}_{t+1,arm<frm}\cdot r^d_{t+1}+\mathcal{S}^{B,(d,frm),nr}_{t+1,arm<frm}\cdot(1-\lambda^{frm})\cdot r^d_{t+1}+\mathcal{S}^{B,d,nr}_{t+1,arm<frm}\cdot(1-\lambda^{frm}-\lambda^{arm})\cdot r^d_{t+1}$$

$$+\mathcal{S}^{B,nd,nr}_{t+1,frm<arm}\cdot r^d_{t+1}+\mathcal{S}^{B,(d,arm),nr}_{t+1,frm<arm}\cdot(1-\lambda^{arm})\cdot r^d_{t+1}+\mathcal{S}^{B,d,nr}_{frm<arm}\cdot(1-\lambda^{frm}-\lambda^{arm})\cdot r^d_{t+1}\Bigg]+\mu^{B,r}_t=1 \tag{112}$$

For the endowment asset we use all the $SDFs$, since the borrower holds this asset whether she refinanced or not and whether she defaulted or not. Finally, the pricing functions are featured in this FOC at in the refinancing states, since the endowment asset affects the mortgage price offered to the borrower.

3. FOC for $\hat{n}_t^B$:.

$$
\begin{aligned}
&E_t\Bigg[\mathcal{S}^{B,nd,r}_{t+1,arm<frm}\Bigg(y^B_{t+1}\cdot(1-\tau^B_{t+1})+p^{B,n}_{t+1}-\hat{m}^{r,frm}_t\left(\frac{\partial\iota^{frm}(\cdot)}{\partial\hat{n}^B_t}\right)\cdot(1+\delta\cdot p^{B,I,frm}_{t+1})\\
&-\hat{m}^{r,arm}_t\left(\frac{\partial spread^{arm}(\cdot)}{\partial\hat{n}^B_t}\right)\cdot(1+\delta\cdot p^{B,I,arm}_{t+1})\Bigg)\\
&+\mathcal{S}^{B,(d,frm),r}_{t+1,arm<frm}\left((1-\lambda^{frm})\cdot(y^B_{t+1}+p^{B,n}_{t+1})-\tau^B_{t+1}\cdot y^B_{t+1}-\hat{m}^{r,arm}_t\left(\frac{\partial spread^{arm}(\cdot)}{\partial\hat{n}^B_t}\right)\cdot(1+\delta\cdot p^{B,I,arm}_{t+1})\right)\\
&+\mathcal{S}^{B,d,r}_{t+1,arm<frm}\left((1-\lambda^{frm}-\lambda^{arm})\cdot(y^B_{t+1}+p^{B,n}_{t+1})-\tau^B_{t+1}\cdot y^B_{t+1}\right)\\
&+\mathcal{S}^{B,nd,r}_{t+1,frm<arm}\Bigg(y^B_{t+1}\cdot(1-\tau^B_{t+1})+p^{B,n}_{t+1}-\hat{m}^{r,frm}_t\left(\frac{\partial\iota^{I,frm}(\cdot)}{\partial\hat{n}^B_t}\right)\cdot(1+\delta\cdot p^{B,I,frm}_{t+1})\\
&-\hat{m}^{r,arm}_t\left(\frac{\partial spread^{arm}(\cdot)}{\partial\hat{n}^B_t}\right)\cdot(1+\delta\cdot p^{B,I,arm}_{t+1})\Bigg)\\
&+\mathcal{S}^{B,(d,arm),r}_{t+1,frm<arm}\left((1-\lambda^{arm})\cdot(y^B_{t+1}+p^{B,n}_{t+1})-\tau^B_{t+1}\cdot y^B_{t+1}-\hat{m}^{r,frm}_t\left(\frac{\partial\iota^{I,frm}(\cdot)}{\partial\hat{n}^B_t}\right)\cdot(1+\delta\cdot p^{B,I,frm}_{t+1})\right)\\
&+\mathcal{S}^{B,d,r}_{frm<arm}\left((1-\lambda^{frm}-\lambda^{arm})\cdot(y^B_{t+1}+p^{B,n}_{t+1})-\tau^B_{t+1}\cdot y^B_{t+1}\right)\Bigg]\\
&E_t\Bigg[\mathcal{S}^{B,nd,nr}_{t+1,arm<frm}\cdot\left(y^B_{t+1}\cdot(1-\tau^B_{t+1})+p^{B,n}_{t+1}\right)+\mathcal{S}^{B,(d,frm),nr}_{t+1,arm<frm}\left((1-\lambda^{frm})\cdot(y^B_{t+1}+p^{B,n}_{t+1})-\tau^B_{t+1}\cdot y^B_{t+1}\right)\\
&+\mathcal{S}^{B,d,nr}_{t+1,arm<frm}\left((1-\lambda^{frm}-\lambda^{arm})\cdot(y^B_{t+1}+p^{B,n}_{t+1})-\tau^B_{t+1}\cdot y^B_{t+1}\right)\\
&+\mathcal{S}^{B,nd,nr}_{t+1,frm<arm}\cdot(y^B_{t+1}\cdot(1-\tau^B_{t+1})+p^{B,n}_{t+1})+\mathcal{S}^{B,(d,arm),nr}_{t+1,frm<arm}\left((1-\lambda^{arm})\cdot(y^B_{t+1}+p^{B,n}_{t+1})-\tau^B_{t+1}\cdot y^B_{t+1}\right)\\
&+\mathcal{S}^{B,d,nr}_{t+1,frm<arm}\left((1-\lambda^{frm}-\lambda^{arm})\cdot(y^B_{t+1}+p^{B,n}_{t+1})-\tau^B_{t+1}\cdot y^B_{t+1}\right)\Bigg]=p^{B,n}_t \qquad (113)
\end{aligned}
$$

For the housing asset we have to use all the $SDFs$ for which there is no total default, we need to account for the fact that under partial default some housing is still held by the borrower. Furthermore, the borrower holds this asset whether she refinanced, so the future returns are affected in both situations. Finally, the pricing functions are featured in this FOC at the refinancing states, since the housing asset affects the mortgage price offered to the borrower.

4. FOC for $\hat{h}_t^B$: alternative.

$$
\begin{aligned}
&E_t\Bigg[\mathcal{S}^{B,nd,r}_{t+1,arm<frm}\Bigg((1-\delta^h)\cdot p^{B,h}_{t+1}\cdot\epsilon_{t+1} - \hat{m}_t^{r,frm}\left(\frac{\partial\iota^{frm}(\cdot)}{\partial\hat{h}_t^B}\right)\cdot(1+\delta\cdot p^{B,I,frm}_{t+1})\\
&-\hat{m}_t^{r,arm}\left(\frac{\partial spread^{arm}(\cdot)}{\partial\hat{h}_t^B}\right)\cdot(1+\delta\cdot p^{B,I,arm}_{t+1})\Bigg)\\
&+\mathcal{S}^{B,(d,frm),r}_{t+1,arm<frm}\left((1-\delta^h)\cdot p^{B,h}_{t+1}\cdot\epsilon_{t+1}\cdot\chi_t^{r,arm} - \hat{m}_t^{r,arm}\left(\frac{\partial spread^{arm}(\cdot)}{\partial\hat{h}_t^B}\right)\cdot(1+\delta\cdot p^{B,I,arm}_{t+1})\right)\\
&+\mathcal{S}^{B,nd,r}_{t+1,frm<arm}\Bigg((1-\delta^h)\cdot p^{B,h}_{t+1}\cdot\epsilon_{t+1} - \hat{m}_t^{r,frm}\left(\frac{\partial\iota^{frm}(\cdot)}{\partial\hat{h}_t^B}\right)\cdot(1+\delta\cdot p^{B,I,frm}_{t+1})\\
&-\hat{m}_t^{r,arm}\left(\frac{\partial spread^{arm}(\cdot)}{\partial\hat{h}_t^B}\right)\cdot(1+\delta\cdot p^{B,I,arm}_{t+1})\Bigg)\\
&+\mathcal{S}^{B,(d,arm),r}_{t+1,frm<arm}\left((1-\delta^h)\cdot p^{B,h}_{t+1}\cdot\epsilon_{t+1}\cdot\chi_t^{r,frm} - \hat{m}_t^{r,frm}\left(\frac{\partial\iota^{frm}(\cdot)}{\partial\hat{h}_t^B}\right)\cdot(1+\delta\cdot p^{B,I,frm}_{t+1})\right)\Bigg]\\
&+E_t\Bigg[\mathcal{S}^{B,nd,nr}_{t+1,arm<frm}(1-\delta^h)\cdot p^{B,h}_{t+1}\cdot\epsilon_{t+1} + \mathcal{S}^{B,(d,frm),nr}_{t+1,arm<frm}(1-\delta^h)\cdot p^{B,h}_{t+1}\cdot\epsilon_{t+1}\cdot\chi_t^{nr,arm}\\
&+\mathcal{S}^{B,nd,nr}_{t+1,frm<arm}(1-\delta^h)\cdot p^{B,h}_{t+1}\cdot\epsilon_{t+1} + \mathcal{S}^{B,(d,arm),nr}_{t+1,frm<arm}(1-\delta^h)\cdot p^{B,h}_{t+1}\cdot\epsilon_{t+1}\cdot\chi_t^{nr,frm}\Bigg] = p_t^{B,h} \qquad (114)
\end{aligned}
$$

For the outstanding mortgage balances assets (both for ARMs and FRMs) we have to use all the $SDFs$ for which there is no total default, we need to account for the fact that under partial default one of the two mortgages types are still held by the borrower. Finally, the pricing functions are not featured in these $FOCs$ since this is debt coming from periods before t. It is worth mentioning that the variable rate FOC takes into account the effect of the interest rate movement in period $t+1$.

5. FOC for $\hat{m}_t^{nr,frm}$:.

$$
\begin{aligned}
&E_t\Bigg[\mathcal{S}^{B,nd,nr}_{t+1,arm<frm}\cdot\left((1-\delta)+\delta\cdot p^{B,m,frm}_{t+1}\right) - \mathcal{S}^{B,(d,frm),nr}_{t+1,arm<frm}\cdot(1-\delta^h)\cdot p^{B,h}_{t+1}\cdot\epsilon_{t+1}\hat{h}_t^B\left(\frac{\partial\chi_t^{nr,arm}}{\partial\hat{m}_t^{nr,frm}}\right)\\
&+\mathcal{S}^{B,nd,nr}_{t+1,frm<arm}\cdot\left((1-\delta)+\delta\cdot p^{B,m,frm}_{t+1}\right)\\
&-\mathcal{S}^{B,(d,arm),nr}_{t+1,frm<arm}\cdot\left[(1-\delta^h)\cdot p^{B,h}_{t+1}\cdot\epsilon_{t+1}\hat{h}_t^B\left(\frac{\partial\chi_t^{nr,frm}}{\partial\hat{m}_t^{nr,frm}}\right) - \left((1-\delta)+\delta\cdot p^{B,m,frm}_{t+1}\right)\right]\Bigg]\\
&+\mu_t^{B,r} = p_t^{B,m,frm} \qquad (115)
\end{aligned}
$$

6. FOC for $\hat{m}_t^{nr,arm}$:.

$$
\begin{aligned}
&\cdot E_t\Bigg[\mathcal{S}^{B,nd,nr}_{t+1,arm<frm}\cdot\Big(\theta^m\cdot i_{t+1}\cdot(1+\delta\cdot p^{B,I,arm}_{t+1})+(1-\delta)+\delta\cdot p^{B,m,arm}_{t+1}\Big)\\
&-\mathcal{S}^{B,(d,frm),nr}_{t+1,arm<frm}\Bigg[(1-\delta^h)\cdot p^{B,h}_{t+1}\cdot\epsilon_{t+1}\hat{h}^B_t\left(\frac{\partial\chi^{nr,arm}_t}{\partial\hat{m}^{nr,arm}_t}\right)\\
&-\Big(\theta^m\cdot i_{t+1}\cdot(1+\delta\cdot p^{B,I,arm}_{t+1})+(1-\delta)+\delta\cdot p^{B,m,arm}_{t+1}\Big)\Bigg]\\
&+\mathcal{S}^{B,nd,nr}_{t+1,frm<arm}\cdot\Big(\theta^m\cdot i_{t+1}\cdot(1+\delta\cdot p^{B,I,arm}_{t+1})+(1-\delta)+\delta\cdot p^{B,m,arm}_{t+1}\Big)\\
&-\mathcal{S}^{B,(d,arm),nr}_{t+1,frm<arm}\cdot(1-\delta^h)\cdot p^{B,h}_{t+1}\cdot\epsilon_{t+1}\hat{h}^B_t\left(\frac{\partial\chi^{nr,frm}_t}{\partial\hat{m}^{nr,arm}_t}\right)\Bigg]\\
&+\mu^{B,r}_t=p^{B,m,arm}_t
\end{aligned}
\tag{116}
$$

For the mortgage interest payment assets (for the ARMs this is the interest spread payment asset) I have to use all the $SDFs$ for which there is no total default, we need to account for the fact that under partial default one of the two mortgages types are still held by the borrower. Finally, the pricing functions are not featured in these $FOCs$ since these are payments coming from periods before t.

7. FOC for $\hat{m}_t^{I,frm}$:.

$$
\begin{aligned}
&E_t\Bigg[\mathcal{S}^{B,nd,nr}_{t+1,arm<frm}\cdot(1+\delta\cdot p^{B,I,frm}_{t+1})\\
&+\mathcal{S}^{B,nd,nr}_{t+1,frm<arm}\cdot(1+\delta\cdot p^{B,I,frm}_{t+1})+\mathcal{S}^{B,(d,arm),nr}_{t+1,frm<arm}\cdot(1+\delta\cdot p^{B,I,frm}_{t+1})\Bigg]=p^{B,I,frm}_t
\end{aligned}
\tag{117}
$$

8. FOC for $\hat{m}_t^{S,arm}$:.

$$
\begin{aligned}
&E_t\Bigg[\mathcal{S}^{B,nd,nr}_{t+1,arm<frm}\cdot(1+\delta\cdot p^{B,I,arm}_{t+1})+\mathcal{S}^{B,(d,frm),nr}_{t+1,arm<frm}\cdot(1+\delta\cdot p^{B,I,arm}_{t+1})\\
&+\mathcal{S}^{B,nd,nr}_{t+1,frm<arm}\cdot(1+\delta\cdot p^{B,I,arm}_{t+1})\Bigg]=p^{B,I,arm}_t
\end{aligned}
\tag{118}
$$

For the newly originated mortgages (both for ARMs and FRMs) I have to use all the $SDFs$ for which there is no total default, we need to account for the fact that under partial default one of the two mortgages types are still held by the borrower. Finally, the pricing functions are featured in this $FOCs$, since it is precisely the newly originated assets that affect the mortgage price offered to the borrower.

9. FOC for $\hat{m}_t^{r,frm}$:.

$$
\begin{aligned}
&E_t\Bigg[\mathcal{S}^{B,nd,r}_{t+1,arm<frm}\Bigg(\left[\iota^{frm}(\cdot)+\hat{m}_t^{r,frm}\left(\frac{\partial\iota^{frm}(\cdot)}{\partial\hat{m}_t^{r,frm}}\right)\right]\cdot(1+\delta\cdot p^{B,I,frm}_{t+1})\\
&+(1-\delta)+\delta\cdot p^{B,m,frm}_{t+1}+\hat{m}_t^{r,arm}\left(\frac{\partial spread^{arm}(\cdot)}{\partial\hat{m}_t^{r,frm}}\right)\cdot(1+\delta\cdot p^{B,I,arm}_{t+1})\Bigg)\\
&-\mathcal{S}^{B,(d,frm),r}_{t+1,arm<frm}\Bigg((1-\delta^h)\cdot p^{B,h}_{t+1}\cdot\epsilon_{t+1}\hat{h}^B_t\left(\frac{\partial\chi_t^{r,arm}}{\hat{m}_t^{r,frm}}\right)-\hat{m}_t^{r,arm}\left(\frac{\partial spread^{arm}(\cdot)}{\partial\hat{m}_t^{r,frm}}\right)\cdot(1+\delta\cdot p^{B,I,arm}_{t+1})\Bigg)\\
&+\mathcal{S}^{B,nd,r}_{t+1,frm<arm}\Bigg(\left[\iota^{frm}(\cdot)+\hat{m}_t^{r,frm}\left(\frac{\partial\iota^{frm}(\cdot)}{\partial\hat{m}_t^{r,frm}}\right)\right]\cdot(1+\delta\cdot p^{B,I,frm}_{t+1})\\
&+(1-\delta)+\delta\cdot p^{B,m,frm}_{t+1}+\hat{m}_t^{r,arm}\left(\frac{\partial spread^{arm}(\cdot)}{\partial\hat{m}_t^{r,frm}}\right)\cdot(1+\delta\cdot p^{B,I,arm}_{t+1})\Bigg)\\
&-\mathcal{S}^{B,(d,arm),r}_{t+1,frm<arm}\Bigg((1-\delta^h)\cdot p^{B,h}_{t+1}\cdot\epsilon_{t+1}\hat{h}^B_t\left(\frac{\partial\chi_t^{r,frm}}{\hat{m}_t^{r,frm}}\right)\\
&-\left[\iota^{frm}(\cdot)+\hat{m}_t^{r,frm}\left(\frac{\partial\iota^{frm}(\cdot)}{\partial\hat{m}_t^{r,frm}}\right)\right](1+\delta\cdot p^{B,I,frm}_{t+1})+(1-\delta)+\delta\cdot p^{B,m,frm}_{t+1}\Bigg)\Bigg]\\
&=\mu_t^{B,r}\left[1-C^m(\cdot)-\hat{m}_t^{r,frm}\left(\frac{\partial C^m(\cdot)}{\partial\hat{m}_t^{r,frm}}\right)\right]
\end{aligned}
\tag{119}
$$

10. FOC for $\hat{m}_t^{r,arm}$:.

$$
\begin{aligned}
& E_t\Bigg[\mathcal{S}^{B,nd,r}_{t+1,arm<frm}\Bigg(\Bigg[spread^{arm}(\cdot)+\hat{m}_t^{r,arm}\left(\frac{\partial spread^{arm}(\cdot)}{\partial \hat{m}_t^{r,arm}}\right)\Bigg](1+\delta\cdot p^{B,I,arm}_{t+1})\\
& +(1-\delta)+\delta\cdot p^{B,m,arm}_{t+1}+\theta^m\cdot i_{t+1}+\hat{m}_t^{r,frm}\left(\frac{\partial \iota^{frm}(\cdot)}{\partial \hat{m}_t^{r,arm}}\right)(1+\delta\cdot p^{B,I,frm}_{t+1})\Bigg)\\
& -\mathcal{S}^{B,(d,frm),r}_{t+1,arm<frm}\Bigg((1-\delta^h)\cdot p^{B,h}_{t+1}\cdot\epsilon_{t+1}\hat{h}^B_t\left(\frac{\partial \chi_t^{r,arm}}{\hat{m}_t^{r,arm}}\right)-(1-\delta)-\delta\cdot p^{B,m,arm}_{t+1}\\
& -\theta^m\cdot i_{t+1}-\Bigg[spread^{arm}(\cdot)+\hat{m}_t^{r,arm}\left(\frac{\partial spread^{arm}(\cdot)}{\partial \hat{m}_t^{r,arm}}\right)\Bigg](1+\delta\cdot p^{B,I,arm}_{t+1})\Bigg)\\
& +\mathcal{S}^{B,nd,r}_{t+1,frm<arm}\Bigg(\Bigg[spread^{arm}(\cdot)+\hat{m}_t^{r,arm}\left(\frac{\partial spread^{arm}(\cdot)}{\partial \hat{m}_t^{r,arm}}\right)\Bigg](1+\delta\cdot p^{B,I,arm}_{t+1})\\
& +(1-\delta)+\delta\cdot p^{B,m,arm}_{t+1}+\theta^m\cdot i_{t+1}+\hat{m}_t^{r,frm}\left(\frac{\partial \iota^{frm}(\cdot)}{\partial \hat{m}_t^{r,arm}}\right)(1+\delta\cdot p^{B,I,frm}_{t+1})\Bigg)\\
& -\mathcal{S}^{B,(d,arm),r}_{t+1,frm<arm}\Bigg((1-\delta^h)\cdot p^{B,h}_{t+1}\cdot\epsilon_{t+1}\hat{h}^B_t\left(\frac{\partial \chi_t^{r,frm}}{\partial \hat{m}_t^{r,arm}}\right)-\hat{m}_t^{r,frm}\left(\frac{\partial \iota^{frm}(\cdot)}{\partial \hat{m}_t^{r,arm}}\right)(1+\delta\cdot p^{B,I,frm}_{t+1})\Bigg)\Bigg]\\
& =\mu_t^{B,r}\left[1-C^m(\cdot)-\hat{m}_t^{r,arm}\left(\frac{\partial C^m(\cdot)}{\partial \hat{m}_t^{r,arm}}\right)\right]
\end{aligned}
\tag{120}
$$

7.13 Saver's Problem

This section resembles the borrower's section 7.9 in this Appendix, so I will summarize it as much as possible. It solves problem (27) presented in section 3.6.2.

7.13.1 Saver's Problem Characterization

Similar to **Proposition 1** in the Appendix 7.9, I can show that the saver's value function (27) can be written as

$$
V^S(w_t^S,\mathcal{Z}_t)=v^S(\mathcal{Z}_t)\cdot\frac{(w_t^S)^{1-\gamma}}{1-\gamma}
$$

where $v^S(\mathcal{Z}_t)$ is a function that will only depend on aggregate state variables. Define the savings (before the portfolio choice) of the saver as,

$$
\Sigma_t^S=w_t^S-c_t^S-\rho_t^{S,h}s_t^S \implies c_t^S+\rho_t^{S,h}s_t^S=w_t^S-\Sigma_t^S.
$$

Using the budget constraint we can write,

$$\Sigma_t^S = p_t^{S,h} h_t^S + p^{S,n} n_t^S + p^{S,\xi} b_t^{S,\xi} + d_t^S \tag{121}$$

We can define the per unit of saving variables, $\hat{x}_t$,

$$\hat{x}_t = \frac{x_t}{\Sigma_t^S}.$$

Using the per unit of saving variables, the budget constraint expression(121) can be rewritten as,

$$1 = p_t^{S,h} \hat{h}_t^S + p^{S,n} \hat{n}_t^S + p_t^{S,\xi} \hat{b}_t^{S,\xi} + \hat{d}_t^S, \tag{122}$$

and tomorrow's wealth (29) can be written as,

$$\hat{w}_{t+1}^S \equiv \frac{\hat{w}_{t+1}^S}{\Sigma_t^S} = (1-\delta^h) p_{t+1}^h \cdot \hat{h}_t^S + \left(y_{t+1}^S + p_{t+1}^{S,n} \right) \hat{n}_t^S + \left(p_{t+1}^{S,\xi} + x_{t+1}^{\xi} \right) \hat{b}_t^{S,\xi} + (1 + r_{t+1}^d) \hat{d}_t^S \tag{123}$$

Conjecture and Verify Procedure The usual results for Cobb-Douglas utility functions imply that the optimal expenditure on non-durable and housing services consumption are

$$\begin{aligned} c_t^S &= \theta^S \cdot \left(w_t^S - \Sigma_t^S \right) \\ s_t^S &= \frac{(1-\theta^S)}{\rho_t^{S,h}} \cdot \left(w_t^S - \Sigma_t^S \right) \end{aligned} \tag{124}$$

The next step is to conjecture and then verify that the value function has the form:

$$V^S(w_t^S, \mathcal{Z}_t) = v^S(\mathcal{Z}_t) \frac{(w_t^S)^{1-\gamma}}{1-\gamma}$$

Plug those two solutions (124) into the saver's Bellman Equation (27),

$$V^S(w_t^S, \mathcal{Z}_t) = \max_{\Sigma_t^S} \frac{C^S(\mathcal{Z}_t)}{1-\gamma} \left(w_t^S - \Sigma_t^S \right)^{1-\gamma} + \left(\Sigma_t^S \right)^{1-\gamma} W^S(\mathcal{Z}_t)$$

where we defined the portfolio choice problem per dollar of savings, $W^S(\mathcal{Z}_t)$,

$$W^S(\mathcal{Z}_t) = \max_{\{\alpha_t^S\}} \quad \psi^S \cdot \frac{(\hat{d}_t^S)^{1-\gamma}}{1-\gamma} + \beta E_t \left[v^S(\mathcal{Z}_{t+1}) \frac{(\hat{w}_t^S)^{1-\gamma}}{1-\gamma} \right] \tag{125}$$

subject to (122). The constant term is: $C^S(\mathcal{Z}_t) = \left((\theta^S)^{\theta^S} + \left(\frac{1-\theta^S}{\rho_t^{S,h}} \right)^{1-\theta^S} \right)^{1-\gamma}$ The value function can then be written as,

$$V^S(w_t^S, \mathcal{Z}_t) = \max_{\Sigma_t^S} \frac{C^S(\mathcal{Z}_t)}{1-\gamma} \left(w_t^S - \Sigma_t^S \right)^{1-\gamma} + \left(\Sigma_t^S \right)^{1-\gamma} W^S(\mathcal{Z}_t) \tag{126}$$

Furthermore, the optimal value for Σ_t^S can be computed as the first order condition of (126),

$$\Sigma_t^S = \frac{\left[(1-\gamma) \cdot W^S(\mathcal{Z}_t) \right]^{\frac{1}{\gamma}}}{C^S(\mathcal{Z}_t)^{\frac{1}{\gamma}} + \left[(1-\gamma) \cdot W^S(\mathcal{Z}_t) \right]^{\frac{1}{\gamma}}} \cdot w_t^S \tag{127}$$

where the fraction of wealth saved is defined as,

$$\sigma_t^S = \frac{\left[(1-\gamma) \cdot W^S(\mathcal{Z}_t) \right]^{\frac{1}{\gamma}}}{C^S(\mathcal{Z}_t)^{\frac{1}{\gamma}} + \left[(1-\gamma) \cdot W^S(\mathcal{Z}_t) \right]^{\frac{1}{\gamma}}}$$

Plug-in the Σ_t^S solution into the Bellman equation.

$$V^S(w_t^S, \mathcal{Z}_t) = \frac{(w_t^S)^{1-\gamma}}{1-\gamma} \left[C^S(\mathcal{Z}_t)(1-\sigma_t^S)^{1-\gamma} + (1-\gamma) \cdot W^S(\mathcal{Z}_t)(\sigma_t^S)^{1-\gamma} \right]$$

which verifies the conjecture of the functional form of $V^S(w_t^S, \mathcal{Z}_t)$. We just need to set

$$v(\mathcal{Z}_t) = C^S(\mathcal{Z}_t)(1-\sigma_t^S)^{1-\gamma} + (1-\gamma) \cdot W^S(\mathcal{Z}_t)(\sigma_t^S)^{1-\gamma}$$

which is a recursion on $v(\mathcal{Z}_t)$.

7.14 Saver's First Order Conditions

In this section, I derive the first order conditions of the borrower's problem as presented in section 3.6.2. As I did with the borrower, I can define saver's wealth in period $t+1$ per unit of wealth at period t can be written as follows using (127),

$$\frac{w_{t+1}^S}{w_t^S} = \frac{\hat{w}_{t+1}^S \cdot \Sigma_t^S}{w_t^S} = \frac{\hat{w}_{t+1}^S \cdot \sigma_t^S \cdot w_t^S}{w_t^S} = \hat{w}_{t+1}^S \cdot \sigma_t^S$$

7.14.1 Preamble for the FOCs

For the saver's problem (27) I define μ_t^S as the multipler of the budget constraint (28). By the envelope condition, this multiplier can be expressed as,

$$\mu_t^S = \frac{v^S(\mathcal{Z}_t)}{\sigma^S(\mathcal{Z}_t)^{-\gamma}},$$

We need to construct the stochastic discount factor (SDF) of the representative saver, we do this here and later use this definitions in the first order conditions.

$$\mathcal{S}_{t+1}^S = \frac{\beta^S \cdot \left[\sigma^S(\mathcal{Z}_t) \cdot \hat{w}_{t+1}^S\right]^{-\gamma} v^S(\mathcal{Z}_{t+1})}{v^S(\mathcal{Z}_t)}. \tag{128}$$

7.14.2 First Order Conditions (FOCs) for the Saver's Problem

1. FOC for $\hat{d}^S$.

$$\frac{\psi^S \cdot (\sigma^S(\mathcal{Z}_t) \cdot \hat{d}_t^S)^{-\gamma}}{v^S(\mathcal{Z}_t)} + E_t\left[\mathcal{S}_{t+1}^S \cdot (1 + r_{t+1}^d)\right] = 1 \tag{129}$$

2. FOC for $\hat{h}^S$.

$$E_t\left[\mathcal{S}_{t+1}^S \cdot \left(1 - \delta^h\right) p_{t+1}^h\right] = p_t^h \tag{130}$$

3. FOC for $\hat{n}^S$.

$$E_t\left[\mathcal{S}_{t+1}^S \cdot \left(y_{t+1}^S + p_{t+1}^{S,n}\right)\right] = p_t^{S,n} \tag{131}$$

4. FOC for $\hat{b}_t^{S,e}$.

$$E_t\left[\mathcal{S}_{t+1}^S \left(p_{t+1}^{S,\xi} + x_{t+1}^{\xi}\right)\right] = p_t^{S,\xi} \tag{132}$$

7.15 Financial Intermediary - PO Payoff Functions

Let $\epsilon_{t+1}^{minus,l,j} = E[\epsilon|\epsilon < \hat{\epsilon}_{t+1}^{l,j}]$ for $l \in \{nr, r\}$ and $j \in \{frm, arm\}$, be the conditional expected value for the housing risk shock when this shock falls below the default threshold. We will use this compact notation for the defaulters in the following expressions.

ARMs The *per dollar* PO payoff function $\mathcal{PO}^{l,arm}$ is defined as,

$$\begin{aligned}
\mathcal{PO}^{l,arm}(\mathcal{Z}_{t+1}) &= \int_{\hat{\epsilon}_{t+1}^{l,arm}}^{\infty} (1-\delta) + \delta \cdot p_{t+1}^{F,m,arm} \cdot f_{t+1}^{\epsilon}(\epsilon) \\
&+ \int_{0}^{\hat{\epsilon}_{t+1}^{l,arm}} (1-\zeta) \cdot p_{t+1}^{h} \cdot (1-\delta^h) \left(\frac{\epsilon_{t+1} \cdot h_t^B}{m_t^{l,arm}} \right) \chi_t^{l,arm} \cdot f_{t+1}^{\epsilon}(\epsilon) \\
&= \left(1 - F_\epsilon(\hat{\epsilon}_{t+1}^{l,arm})\right) \left[(1-\delta) + \delta \cdot p_{t+1}^{F,m,arm}\right] \\
&+ F_\epsilon(\hat{\epsilon}_{t+1}^{l,arm}) \cdot (1-\zeta) \cdot p_{t+1}^{h} \cdot (1-\delta^h) \left(\frac{\epsilon_{t+1}^{minus,l,arm} \cdot h_t^B}{m_t^{l,arm}} \right) \chi_t^{l,arm}
\end{aligned} \tag{133}$$

Equation (133) shows the ARMs payoff function for the *principal-only* asset for any refinancing state $l \in \{r, nr\}$. The first term represents the fraction of borrowers that end up repaying their debt times the value of that repayment per dollar lent in period t. The second term represent the fraction $(1-\zeta)$ of the housing value that is lost when a house of a defaulting borrower is repossessed. Notice that only the borrowers with the lowest ϵ_{t+1} default and $\chi_t^{l,arm}$ appears in this equation because the ARMs are backed only by this fraction of the housing. The first component of the repayment term refers to the cash flow that is entitled to the holder of the PO asset per unit of dollar, and the second term represent the per unit resale value equals $\delta \cdot p_{t+1}^{F,m,arm}$ at the following trading stage.

FRMs The *per dollar* continuation function $\mathcal{PO}^{l,frm}$ is defined as,

$$\begin{aligned}
\mathcal{PO}^{l,frm}(\mathcal{Z}_{t+1}) &= (1-s_t)\Bigg(\int_{\hat{\epsilon}_{t+1}^{l,frm}}^{\infty}(1-\delta)+\delta\cdot p_{t+1}^{F,m,frm}\cdot f_{t+1}^{\epsilon}(\epsilon) \\
&\quad+\int_{0}^{\hat{\epsilon}_{t+1}^{l,frm}}(1-\zeta)\cdot p_{t+1}^{h}\cdot(1-\delta^{h})\left(\frac{\epsilon_{t+1}\cdot h_t^B}{m_t^{l,frm}}\right)\chi_t^{l,frm}\cdot f_{t+1}^{\epsilon}(\epsilon)\Bigg) \\
&\quad+s_t\left(\int_{\hat{\epsilon}_{t+1}^{l,frm}}^{\infty}(1-\delta)+\delta\cdot p_{t+1}^{F,m,frm}\cdot f_{t+1}^{\epsilon}(\epsilon)+\int_{0}^{\hat{\epsilon}_{t+1}^{l,frm}}1\cdot f_{t+1}^{\epsilon}(\epsilon)\right) \\
&= (1-s_t)\Bigg(\left(1-F_{\epsilon}(\hat{\epsilon}_{t+1}^{l,frm})\right)\left[(1-\delta)+\delta\cdot p_{t+1}^{F,m,frm}\right] \\
&\quad F_{\epsilon}(\hat{\epsilon}_{t+1}^{l,frm}\cdot(1-\zeta)\cdot p_{t+1}^{h}\cdot(1-\delta^{h})\left(\frac{\epsilon_{t+1}^{minus,l,frm}\cdot h_t^B}{m_t^{l,frm}}\right)\chi_t^{l,frm}\Bigg) \\
&\quad+s_t\left[\left(1-F_{\epsilon}(\hat{\epsilon}_{t+1}^{l,frm})\right)\left((1-\delta)+\delta\cdot p_{t+1}^{F,m,frm}\right)+F_{\epsilon}(\hat{\epsilon}_{t+1}^{l,frm})\cdot 1\right]
\end{aligned} \tag{134}$$

Equation (134) shows the FRMs payoff function for the *principal-only* asset for any refinancing state $l \in \{r, nr\}$. This payoff function differs from (133) since FRMs can be insured. The terms multiplying $(1-s_t)$ represent the non-insured part of the FRMs balance sheet, while the terms multiplying s_t represent the insured part of the FRMs balance sheet.

Terms multiplying $(1-s_t)$ The first term represents the fraction of borrowers that end up repaying their debt times the value of that repayment per dollar lent in period t. The second term represent the fraction $(1-\zeta)$ of the housing value that is lost when a house of a defaulting borrower is repossessed. Notice that only the borrowers with the lowest ϵ_{t+1} default and $\chi_t^{l,frm}$ appears in this equation because the ARMs are backed only by this fraction of the housing. These are the same terms observed in the ARMs payoff function. The first component of the repayment term refers to the cash flow that is entitled to the holder of the IO asset per unit of dollar, and the second term represent the per unit resale value equals $\delta \cdot p_{t+1}^{F,m,frm}$ at the following trading stage.

Terms multiplying s_t The first term represents the fraction of borrowers that end up repaying their debt times the value of that repayment per dollar lent in period t. This is exactly the same expression than the one observed for the non-insured fraction. The big difference comes from those defaulted and insured mortgages, in this case the financial intermediary only receives the mortgage payment but no the resale price (last term in equation (134)). When defaulted the mortgage exits the balance sheet even when insured, the government insurance pays back the face value of the mortgage and the interest payment due in the defaulted period (see equation (37)).

7.16 Capital Requirement Equations

In this section I show how to compute the expression for (45) and (46). The *per dollar* IO payoff function $\mathcal{IO}^{l,frm,uninsured}$ for uninsured FRMs is defined as,

$$\mathcal{IO}^{l,frm,uninsured}(\mathcal{Z}_{t+1}, m_t^{l,frm}) = (1-s_t)\Bigg[\left(1 - F_\epsilon(\hat{\epsilon}_{t+1}^{l,frm})\right)\left(\frac{\text{payment}_{t+1}^{l,frm}(1+\delta\cdot p_{t+1}^{F,I,frm})}{m_t^{l,frm}}\right) + F_\epsilon(\hat{\epsilon}_{t+1}^{l,frm})\cdot 0\Bigg] \tag{135}$$

The *per dollar* IO payoff function $\mathcal{IO}^{l,frm,insured}$ for insured FRMs is defined as,

$$\mathcal{IO}^{l,frm,insured}(\mathcal{Z}_{t+1}, m_t^{l,frm}) = s_t\Bigg[\left(1 - F_\epsilon(\hat{\epsilon}_{t+1}^{l,frm})\right)\left(\frac{\text{payment}_{t+1}^{l,frm}(1+\delta\cdot p_{t+1}^{F,I,frm})}{m_t^{l,frm}}\right) + F_\epsilon(\hat{\epsilon}_{t+1}^{l,frm})\left(\frac{\text{payment}_{t+1}^{l,frm}}{m_t^{l,frm}}\right)\Bigg] \tag{136}$$

The *per dollar* PO payoff function $\mathcal{PO}^{l,frm,uninsured}$ for uninsured FRMs is defined as,

$$\mathcal{PO}^{l,frm,uninsured}(\mathcal{Z}_{t+1}) = (1-s_t)\Bigg[\left(1 - F_\epsilon(\hat{\epsilon}_{t+1}^{l,frm})\right)\left[(1-\delta) + \delta\cdot p_{t+1}^{F,m,frm}\right] F_\epsilon(\hat{\epsilon}_{t+1}^{l,frm}\cdot(1-\zeta)\cdot p_{t+1}^h\cdot(1-\delta^h)\left(\frac{\epsilon_{t+1}^{minus,l,frm}\cdot h_t^B}{m_t^{l,frm}}\right)\chi_t^{l,frm}\Bigg] \tag{137}$$

The *per dollar* PO payoff function $\mathcal{PO}^{l,frm,insured}$ for insured FRMs is defined as,

$$\mathcal{PO}^{l,frm,insured}(\mathcal{Z}_{t+1}) = s_t\Bigg[\left(1 - F_\epsilon(\hat{\epsilon}_{t+1}^{l,frm})\right)\left((1-\delta) + \delta\cdot p_{t+1}^{F,m,frm}\right) + F_\epsilon(\hat{\epsilon}_{t+1}^{l,frm})\cdot 1\Bigg] \tag{138}$$

The *aggregate* payoff function for uninsured FRMs conditional on the refinancing status $l \in \{r, nr\}$ in period t is defined as,

$$\mathcal{P}_{agg}^{l,frm,uninsured}(\mathcal{Z}_{t+1}) = \left(\mathcal{IO}^{l,frm,uninsured}(\mathcal{Z}_{t+1}, m_t^{l,frm}) + \mathcal{PO}^{l,frm,uninsured}(\mathcal{Z}_{t+1})\right)\cdot M_t^{F,l,frm} \tag{139}$$

Then expression (45) can be computed as,

$$\mathcal{P}_{agg}^{frm,uninsured}(\mathcal{Z}_{t+1}) = \sum_{s=\{nr,r\}} q^s \cdot \mathcal{P}_{agg}^{s,frm,uninsured}(\mathcal{Z}_{t+1})$$

The *aggregate* payoff function for insured FRMs conditional on the refinancing status $l \in \{r, nr\}$ in period t is,

$$\mathcal{P}_{agg}^{l,frm,insured}(\mathcal{Z}_{t+1}) = \left(\mathcal{IO}^{l,frm,insured}(\mathcal{Z}_{t+1}, m_t^{l,frm}) + \mathcal{PO}^{l,frm,insured}(\mathcal{Z}_{t+1})\right) \cdot M_t^{F,l,frm} \tag{140}$$

Then expression (46) can be computed as,

$$\mathcal{P}_{agg}^{frm,insured}(\mathcal{Z}_{t+1}) = \sum_{s=\{nr,r\}} q^s \cdot \mathcal{P}_{agg}^{s,frm,insured}(\mathcal{Z}_{t+1})$$

7.17 Rewriting the Capital Requirement Constraint

The financial intermediary has to satisfy the following set of constraints at time t

$$\begin{aligned} e(\mathcal{Z}_{t+1}) \geq \bar{e}^{frm,insured} \cdot \mathcal{P}_{agg}^{frm,insured}(\mathcal{Z}_{t+1}) + \bar{e}^{frm,uninsured} \cdot \mathcal{P}_{agg}^{frm,uninsured}(\mathcal{Z}_{t+1}) \\ + \bar{e}^{arm} \cdot \mathcal{P}_{agg}^{arm}(\mathcal{Z}_{t+1}), \quad \forall z_{t+1}|\mathcal{Z}_t \end{aligned} \tag{141}$$

or equivalently using the definition of tomorrow's equity (43)

$$\begin{aligned} \left(1 - \bar{e}^{frm,insured}\right) \cdot \mathcal{P}_{agg}^{frm,insured}(\mathcal{Z}_{t+1}) + \left(1 - \bar{e}^{frm,uninsured}\right) \cdot \mathcal{P}_{agg}^{frm,uninsured}(\mathcal{Z}_{t+1}) \\ + (1 - \bar{e}^{arm}) \cdot \mathcal{P}_{agg}^{arm}(\mathcal{Z}_{t+1}) \geq \mathcal{P}^{\text{insurance}} + D_t \cdot (1 + r_{t+1}^d), \quad \forall z_{t+1}|\mathcal{Z}_t \end{aligned} \tag{142}$$

At time t, it suffices to impose the constraint for the worst possible aggregate state in $t+1$: if the solvency condition is binding for the worst possible payoff of the mortgage portfolio, it will be slack for all higher payoff realizations. Notice the right hand side of (142) does not depend on the state tomorrow. This implies that we can define

$$\begin{aligned} \underline{z}_t = \underset{z_{t+1}|\mathcal{Z}_t}{\text{argmin}} \ \left(1 - \bar{e}^{frm,insured}\right) \cdot \mathcal{P}_{agg}^{frm,insured}(\mathcal{Z}_{t+1}) + \left(1 - \bar{e}^{frm,uninsured}\right) \cdot \mathcal{P}_{agg}^{frm,uninsured}(\mathcal{Z}_{t+1}) \\ + (1 - \bar{e}^{arm}) \cdot \mathcal{P}_{agg}^{arm}(\mathcal{Z}_{t+1}) - \mathcal{P}^{\text{insurance}} - D_t \cdot (1 + r_{t+1}^d) \end{aligned}$$

and impose a single constraint at time t

$$\left(1-\bar{e}^{frm,insured}\right)\cdot\mathcal{P}_{agg}^{frm,insured}(\underline{z}_t)+\left(1-\bar{e}^{frm,uninsured}\right)\cdot\mathcal{P}_{agg}^{frm,uninsured}(\underline{z}_t)$$
$$+\left(1-\bar{e}^{arm}\right)\cdot\mathcal{P}_{agg}^{arm}(\underline{z}_t)-\mathcal{P}^{\text{insurance}}-D_t\cdot(1+r_{t+1}^{d})\geq 0. \tag{143}$$

7.18 Solution to the Financial Intermediary's Problem

In this section I solve the problem of the financial intermediary presented in (3.7.8). the intermediary's problem is to optimally choose,

$$\alpha_t^F=\left\{M_t^{F,r,frm},M_t^{F,r,arm},M_t^{F,nr,frm},M_t^{F,nr,arm},M_t^{F,I,frm},M_t^{F,S,arm},s_t,D_t,I_t\right\},$$

to maximize the dividend paid to the owners ($\tau e_t - I_t$). Let μ_t^F be the multiplier on the budget constraint (35), and κ_t^F the multiplier on the leverage constraint (143). The problem of the financial intermediary can be written in Lagrangian form as,

$$V^F(e_t,\mathcal{Z}_t)=\max_{\alpha_t^I}\tau e_t-I_t+\mathrm{E}_t\left[\mathcal{S}_{t,t+1}^{S}\cdot V^I\left(e_{t+1},\mathcal{Z}_{t+1}\right)\right]$$
$$+\kappa_t^F\Bigg[\left(1-\bar{e}^{frm,insurance}\right)\cdot\mathcal{P}_{agg}^{frm,insurance}(\underline{z}_t)+\left(1-\bar{e}^{frm,uninsurance}\right)\cdot\mathcal{P}_{agg}^{frm,uninsurance}(\underline{z}_t)$$
$$+\left(1-\bar{e}^{arm}\right)\cdot\mathcal{P}_{agg}^{arm}(\underline{z}_t)-\mathcal{P}^{\text{insurance}}-D_t\cdot(1+i_t^d)\Bigg]$$
$$+\mu_t^F\Bigg[(1-\tau)\cdot e_t+I_t-C^F(I_t)+D_t+q^r\cdot\sum_{j\in\{frm,arm\}}M_t^{F,nr,j}$$
$$-\bar{M}_t^F-q^r\cdot\sum_{j\in\{frm,arm\}}M_t^{F,r,j}-C^{orig}\left(\sum_{j\in\{frm,arm\}}M_t^{F,r,j}\right)\Bigg] \tag{144}$$

I can compute the **envelope condition** from (144) which represents the marginal value of equity,

$$\frac{\partial V^F(e_t,\mathcal{Z}_t)}{\partial e_t}=\tau+(1-\tau)\mu_t^F \tag{145}$$

Next, I will compute the first order conditions that solves problem (144).

1. FOC for equity issuance I_t:

$$-1+\mu_t^F\left(1-\frac{\partial C^F(I_t)}{\partial I_t}\right)=0, \tag{146}$$

I can use the functional form on $C^F(I_t)$ to find a solution for the budget constraint multiplier μ_t^F,

$$\mu_t^F=\frac{1}{1-\chi\cdot I_t}, \tag{147}$$

and the envelope condition (145) can be written as,

$$\frac{\partial V^F(e_t,\mathcal{Z}_t)}{\partial e_t}=\tau+(1-\tau)\mu_t^F=\tau+\left(\frac{1-\tau}{1-\chi\cdot I_t}\right) \tag{148}$$

$$\tag{149}$$

2. The FOC with respect to deposits, D_t

$$-\mathrm{E}_t\left[\mathcal{S}^S_{t,t+1}\left(\frac{\partial V^F(e_{t+1},\mathcal{Z}_{t+1})}{\partial e_{t+1}}\right)\cdot(1+r^d_{t+1})\right]-\kappa_t^I\cdot(1+r^d_{t+1})+\mu_t^F=0, \tag{150}$$

I can define the intermediary's stochastic discount factor, $\mathcal{S}^F_{t,t+1}$, as

$$\mathcal{S}^F_{t,t+1}=\mathcal{S}^S_{t,t+1}\left(\frac{\partial V^F(e_{t+1},\mathcal{Z}_{t+1})}{\partial e_{t+1}}\right)\cdot\frac{1}{\mu_t^F}=\mathcal{S}^S_{t,t+1}\cdot\left(\tau+\left(\frac{1-\tau}{1-\chi\cdot I_{t+1}}\right)\right)\cdot(1-\chi\cdot I_t), \tag{151}$$

I can rewrite the deposit's first order condition (150) using (151). I will also use (151) for the following first order condition in this section.

$$\mathrm{E}_t\left[\mathcal{S}^F_{t,t+1}\cdot(1+r^d_{t+1})\right]+\kappa_t^F\cdot(1+r^d_{t+1})=1 \tag{152}$$

$$\tag{153}$$

3. The FOC with respect to PO asset for FRMs, $M_t^{F,nr,frm}$

I first define the following auxiliary variables:

$$\upsilon^{m,nr,frm,1}_{t+1}(\mathcal{Z}_{t+1})=F_\epsilon(\hat{\epsilon}^{nr,frm}_{t+1})(1-\zeta)p^h_{t+1}(1-\delta^h)\left(\frac{\epsilon^{minus,nr,frm}_{t+1}\cdot h^B_t}{m_t^{nr,frm}}\right)\chi_t^{nr,frm}$$

$$\upsilon_{t+1}^{m,nr,frm,2}(\mathcal{Z}_{t+1}) = \Big(1 - F_\epsilon(\hat{\epsilon}_{t+1}^{nr,frm})\Big)\Big((1-\delta) + \delta \cdot p_{t+1}^{F,m,frm}\Big)$$

The FOC can be written as,

$$\begin{aligned} p_t^{F,m,frm} &= q^r + q^{nr} \cdot E_t\Bigg[\mathcal{S}_{t,t+1}^F\Bigg\{(1-s_t)\Bigg(\upsilon_{t+1}^{m,nr,frm,1}(\mathcal{Z}_{t+1}) + \upsilon_{t+1}^{m,nr,frm,2}(\mathcal{Z}_{t+1})\Bigg) \\ &+ s_t\Bigg(\upsilon_{t+1}^{m,nr,frm,2}(\mathcal{Z}_{t+1}) + F_\epsilon(\hat{\epsilon}_{t+1}^{nr,frm})\Bigg) - \phi^{fee} \cdot s_t \cdot p_t^{F,m,frm}\Bigg\}\Bigg] \\ &+ q^{nr} \cdot \kappa_t^I\Bigg[(1 - \bar{e}^{frm,uninsured}) \cdot (1-s_t) \cdot \Bigg(\upsilon_{t+1}^{m,nr,frm,1}(\underline{z}_t) + \upsilon_{t+1}^{m,nr,frm,2}(\underline{z}_t)\Bigg) \\ &+ (1 - \bar{e}^{frm,insured}) \cdot s_t \cdot \Bigg(\upsilon_{t+1}^{m,nr,frm,2}(\underline{z}_t) + F_\epsilon(\hat{\epsilon}_{t+1}^{nr,frm}(\underline{z}_t))\Bigg) - \phi^{fee} \cdot s_t \cdot p_t^{I,m,frm}\Bigg] \end{aligned} \tag{154}$$

This first order condition pins down the secondary market price for the PO asset for FRMs, $p_t^{F,m,frm}$, which is assumed to be a competitive market. The purchase of a unit of the PO asset at the trading stage in period t should be equal to the following three elements:

1. In case the intermediary decides not to get insurance it receives the uncertain payment $\upsilon_{t+1}^{m,nr,frm,1}(\mathcal{Z}_{t+1}) + \upsilon_{t+1}^{m,nr,frm,2}(\mathcal{Z}_{t+1})$. On the other hand, in case the intermediary decides to get insurance it receives the uncertain payment $\upsilon_{t+1}^{m,nr,frm,1}(\mathcal{Z}_{t+1}) + F_\epsilon(\hat{\epsilon}_{t+1}^{nr,frm})$. The last term represents a payment equal to the face value in case of default. Insuring comes at a cost of $\phi^{fee} \cdot s_t \cdot p_t^{F,m,frm}$. The intermediary has to consider compute the expected value of such payments discounted at $\mathcal{S}_{t,t+1}^F$. Balancing these effects is an important element of the paper.

2. The effect that the terms described in point 1 have on the risk-weighted capital constraint.

3. In case the borrower refinances in period t, the intermediary fully receives the face value of this old mortgage debt.

4. The FOC with respect to PO asset for ARMs, $M_t^{F,nr,arm}$

I first define the following auxiliary variables:

$$\upsilon_{t+1}^{m,nr,arm,1}(\mathcal{Z}_{t+1}) = F_\epsilon(\hat{\epsilon}_{t+1}^{nr,arm})(1-\zeta)p_{t+1}^h(1-\delta^h)\left(\frac{\epsilon_{t+1}^{minus,nr,arm} \cdot h_t^B}{m_t^{nr,arm}}\right)\chi_t^{nr,arm}$$

$$\upsilon_{t+1}^{m,nr,arm,2}(\mathcal{Z}_{t+1}) = \big(1 - F_\epsilon(\hat{\epsilon}_{t+1}^{nr,arm})\big)\Big((1-\delta) + \delta \cdot p_{t+1}^{F,m,arm}\Big)$$

The FOC can be written as,

$$
\begin{aligned}
p_t^{F,m,arm} &= q^r + q^{nr} \cdot E_t \Bigg[\mathcal{S}_{t,t+1}^F \Bigg\{ \upsilon_{t+1}^{m,nr,arm,1}(\mathcal{Z}_{t+1}) + \upsilon_{t+1}^{m,nr,arm,2}(\mathcal{Z}_{t+1}) \\
&+ \big(1 - F_\epsilon(\hat{\epsilon}_{t+1}^{nr,arm})\big) \cdot i(\mathcal{Z}_{t+1}) \cdot \Big(1 + \delta \cdot p_{t+1}^{I,\iota,arm}\Big) \Bigg\} \Bigg] \\
&+ q^{nr} \cdot \kappa_t^I \Bigg[(1 - \bar{e}^{arm}) \Bigg\{ \upsilon_{t+1}^{m,nr,arm,1}(\underline{z}_t)) + \upsilon_{t+1}^{m,nr,arm,2}(\underline{z}_t)) \\
&+ \big(1 - F_\epsilon(\hat{\epsilon}_{t+1}^{nr,arm}(\underline{z}_t))\big) \cdot i(\underline{z}_t) \cdot p_{t+1}^{I,\iota,arm}(\underline{z}_t) \Bigg\} \Bigg] \qquad (155)
\end{aligned}
$$

This first order condition pins down the secondary market price for the PO asset for ARMs, $p_t^{F,m,arm}$, which is assumed to be a competitive market. The purchase of a unit of the PO asset at the trading stage in period t should be equal to the following three elements:

1. In case the intermediary cannot get insurance for ARMs. It receives the uncertain payment $\upsilon_{t+1}^{m,nr,arm,1}(\mathcal{Z}_{t+1})+$ $\upsilon_{t+1}^{m,nr,arm,2}(\mathcal{Z}_{t+1})$. Additionally, I make the adjustable term explicit, this is what makes this mortgage different from the FRMs, and provides a 'natural' insurance. The intermediary has to consider compute the expected value of such payments discounted at $\mathcal{S}_{t,t+1}^F$.

2. The effect that the terms described in point 1 have on the risk-weighted capital constraint.

3. In case the borrower refinances in period t, the intermediary fully receives the face value of this old mortgage debt.

5. The FOC with respect to the IO asset for FRMs $M_t^{F,I,frm}$:

I first define the following auxiliary variable:

$$
\upsilon_{t+1}^{I,nr,frm,1}(\mathcal{Z}_{t+1}) = (1 - F_\epsilon(\hat{\epsilon}_{t+1}^{nr,frm}))\left(1 + \delta \cdot p_{t+1}^{F,I,frm}\right)
$$

The FOC can be written as,

$$
\begin{aligned}
p_t^{F,I,frm} =& q^{nr} \cdot E_t \Bigg[\mathcal{S}_{t,t+1}^F \Bigg\{ (1-s_t) \Bigg(\upsilon_{t+1}^{I,nr,frm,1}(\mathcal{Z}_{t+1}) \Bigg) \\
&+ s_t \Bigg(\upsilon_{t+1}^{I,nr,frm,1}(\mathcal{Z}_{t+1}) + F_\epsilon(\hat{\epsilon}_{t+1}^{nr,frm}) \Bigg) - \phi^{fee} \cdot s_t \cdot p_t^{F,I,frm} \Bigg\} \Bigg] \\
&+ q^{nr} \cdot \kappa_t^I \Bigg[(1 - e^{frm,uninsured})(1-s_t) \Bigg(\upsilon_{t+1}^{I,nr,frm,1}(\underline{z}_t) \Bigg) \\
&+ (1 - e^{frm,insured}) \cdot s_t \cdot \Bigg(\upsilon_{t+1}^{I,nr,frm,1}(\underline{z}_t) + F_\epsilon(\hat{\epsilon}_{t+1}^{nr,frm}(\underline{z}_t)) \Bigg) - \phi^{fee} \cdot s_t \cdot p_t^{I,\iota,frm} \Bigg]
\end{aligned} \tag{156}
$$

This first order condition pins down the secondary market price for the IO asset for FRMs, $p_t^{F,m,frm}$, which is assumed to be a competitive market. The purchase of a unit of the IO asset at the trading stage in period t should be equal to the following two elements.

1. In case the intermediary decides not to get insurance it receives the uncertain payment $\upsilon_{t+1}^{I,nr,frm,1}(\mathcal{Z}_{t+1})$. On the other hand, in case the intermediary decides to get insurance it receives the uncertain payment $\upsilon_{t+1}^{I,nr,frm,1}(\mathcal{Z}_{t+1}) + F_\epsilon(\hat{\epsilon}_{t+1}^{nr,frm})$. The last term represents a payment equal to the mortgage interest payment due in period $t+1$, which I assume is paid in case of government insurance. Insuring comes at a cost of $\phi^{fee} \cdot s_t \cdot p_t^{F,I,frm}$. The intermediary has to consider compute the expected value of such payments discounted at $\mathcal{S}_{t,t+1}^F$. Balancing these effects is an important element of the paper.

2. The effect that the terms described in point 1 have on the risk-weighted capital constraint.

6. The FOC with respect to the IO asset for ARMs $M_t^{F,S,arm}$:

I first define the following auxiliary variable:

$$
\upsilon_{t+1}^{S,nr,arm,1}(\mathcal{Z}_{t+1}) = (1 - F_\epsilon(\hat{\epsilon}_{t+1}^{nr,arm})) \left(1 + \delta \cdot p_{t+1}^{F,I,arm} \right)
$$

The FOC can be written as,

$$
p_t^{F,I,arm} = E_t \Bigg[\mathcal{S}_{t,t+1}^F \cdot \upsilon_{t+1}^{S,nr,arm,1}(\mathcal{Z}_{t+1}) \Bigg] + \kappa_t^I \Bigg[(1 - \overline{e}^{arm}) \upsilon_{t+1}^{S,nr,arm,1}(\underline{z}_t) \Bigg] \tag{157}
$$

This first order condition pins down the secondary market price for the IO asset for ARMs, $p_t^{F,I,arm}$, which is assumed to be a competitive market. The purchase of a unit of the IO asset at the trading stage in period t should be equal to the following two elements:

1. In case the intermediary cannot get insurance for ARMs. It receives the uncertain payment $\upsilon_{t+1}^{S,nr,arm,1}(\mathcal{Z}_{t+1})$. The intermediary has to consider compute the expected value of such payments discounted at $\mathcal{S}_{t,t+1}^{F}$.

2. The effect that the terms described in point 1 have on the risk-weighted capital constraint.

7. The FOC with respect to the new FRM originations $M_t^{F,r,frm}$:

I first define the following auxiliary variable:

$$\upsilon_{t+1}^{m,r,frm,1}(\mathcal{Z}_{t+1}) = (1 - F_{\epsilon}(\hat{\epsilon}_{t+1}^{r,frm}))\left(1 + \delta \cdot p_{t+1}^{F,I,frm}\right)$$

$$\upsilon_{t+1}^{m,r,frm,2}(\mathcal{Z}_{t+1}) = F_{\epsilon}(\hat{\epsilon}_{t+1}^{r,frm})(1-\zeta) \cdot p_{t+1}^{h} \cdot (1-\delta^{h})\left(\frac{\epsilon_{t+1}^{minus,r,frm} \cdot h_t^B}{m_t^{r,frm}}\right)\chi_t^{r,frm}$$

$$\upsilon_{t+1}^{m,r,frm,3}(\mathcal{Z}_{t+1}) = \left(1 - F_{\epsilon}(\hat{\epsilon}_{t+1}^{r,frm})\right)\left((1-\delta) + \delta \cdot p_{t+1}^{F,m,frm}\right)$$

The FOC can be written as,

$$\begin{aligned}
&E_t\Bigg[\mathcal{S}_{t,t+1}^{F}\Bigg\{(1-s_t)\Bigg(\upsilon_{t+1}^{m,r,frm,1}(\mathcal{Z}_{t+1}) \cdot \iota^{frm}\left(\alpha_t^{B,r}, \mathcal{Z}_t\right) + \upsilon_{t+1}^{m,r,frm,2}(\mathcal{Z}_{t+1}) + \upsilon_{t+1}^{m,r,frm,3}(\mathcal{Z}_{t+1})\Bigg) \\
&+ s_t\Bigg(\upsilon_{t+1}^{m,r,frm,1}(\mathcal{Z}_{t+1}) \cdot \iota^{frm}\left(\alpha_t^{B,r}, \mathcal{Z}_t\right) + \upsilon_{t+1}^{m,r,frm,3}(\mathcal{Z}_{t+1}) + F_{\epsilon}(\hat{\epsilon}_{t+1}^{r,frm}) \cdot \left(1 + \iota^{frm}\left(\alpha_t^{B,r}, \mathcal{Z}_t\right)\right)\Bigg) - \phi^{fee} \cdot s_t\Bigg\}\Bigg] \\
&+ \kappa_t^I\Bigg[\left(1 - \overline{e}^{frm,uninsured}\right)(1-s_t)\Bigg(\upsilon_{t+1}^{m,r,frm,1}(\underline{z}_t) \cdot \iota^{frm}\left(\alpha_t^{B,r}, \underline{z}_t\right) + \upsilon_{t+1}^{m,r,frm,2}(\underline{z}_t) + \upsilon_{t+1}^{m,r,frm,3}(\underline{z}_t)\Bigg) \\
&+ \left(1 - \overline{e}^{frm,insured}\right) \cdot s_t \cdot \Bigg(\upsilon_{t+1}^{m,r,frm,1}(\underline{z}_t) \cdot \iota^{frm}\left(\alpha_t^{B,r}, \underline{z}_t\right) + \upsilon_{t+1}^{m,r,frm,3}(\underline{z}_t) + F_{\epsilon}(\hat{\epsilon}_{t+1}^{r,frm}(\underline{z}_t))\left(1 + \iota^{frm}(\underline{z}_t)\right)\Bigg) \\
&- \phi^{fee} \cdot s_t\Bigg] = \left(1 + \frac{\partial C^{orig}(\cdot)}{\partial M_t^{F,r,frm}}\right)
\end{aligned} \tag{158}$$

8. The FOC with respect to the new FRM originations $M_t^{F,r,arm}$:

I first define the following auxiliary variable:

$$\upsilon_{t+1}^{m,r,arm,1}(\mathcal{Z}_{t+1}) = (1 - F_{\epsilon}(\hat{\epsilon}_{t+1}^{r,arm}))\left(1 + \delta \cdot p_{t+1}^{F,I,arm}\right)$$

$$\upsilon_{t+1}^{m,r,arm,2}(\mathcal{Z}_{t+1}) = F_\epsilon(\hat{\epsilon}_{t+1}^{r,arm})(1-\zeta)\cdot p_{t+1}^h \cdot (1-\delta^h)\left(\frac{\epsilon_{t+1}^{minus,r,arm}\cdot h_t^B}{m_t^{r,arm}}\right)\chi_t^{r,arm}$$

$$\upsilon_{t+1}^{m,r,arm,3}(\mathcal{Z}_{t+1}) = \left(1 - F_\epsilon(\hat{\epsilon}_{t+1}^{r,arm})\right)\left((1-\delta) + \delta\cdot p_{t+1}^{F,m,arm}\right)$$

The FOC can be written as,

$$\begin{aligned}
&E_t\left[\mathcal{S}_{t,t+1}^F\left\{\upsilon_{t+1}^{m,r,arm,1}(\mathcal{Z}_{t+1})\cdot\left(i(\mathcal{Z}_{t+1}) + spread^{arm}(\alpha_t^{B,r},\mathcal{Z}_t)\right) + \upsilon_{t+1}^{m,r,arm,2}(\mathcal{Z}_{t+1}) + \upsilon_{t+1}^{m,r,arm,3}(\mathcal{Z}_{t+1})\right\}\right]\\
&+\kappa_t^I\left[(1-\overline{e}^{arm})\left\{\upsilon_{t+1}^{m,r,arm,1}(\underline{z}_t)\left(i(\underline{z}_t) + spread^{arm}(\alpha_t^{B,r},\underline{z}_t)\right) + \upsilon_{t+1}^{m,r,arm,2}(\underline{z}_t) + \upsilon_{t+1}^{m,r,arm,3}(\underline{z}_t)\right]\\
&= \left(1 + \frac{\partial C^{orig}(\cdot)}{\partial M_t^{F,r,arm}}\right)
\end{aligned}\tag{159}$$

9. The FOC with respect to insured fraction s_t:

This first order condition will just feature FRMs, since the insurance is only provided to this mortgage type, I have to distinguish between the insured and uninsured payoffs introduced in equations (45) and (46), I derived those in detail in Appendix 7.16. The FOC can be written as,

$$\begin{aligned}
&E_t\left[\mathcal{S}_{t,t+1}^F\left\{\mathcal{P}_{agg}^{frm,insured}(\mathcal{Z}_{t+1}) - \mathcal{P}_{agg}^{frm,uninsured}(\mathcal{Z}_{t+1}) - \phi^{fee}\cdot\bar{M}_t^{F,frm}\right\}\right]\\
&+\kappa_t^I\left[(1-\overline{e}^{frm,insured})\cdot\mathcal{P}_{agg}^{frm,insured}(\underline{z}_t) - (1-\overline{e}^{frm,uninsured})\cdot\mathcal{P}_{agg}^{frm,uninsured}(\underline{z}_t) - \phi^{fee}\cdot\bar{M}_t^{F,frm}\right] = 0
\end{aligned}\tag{160}$$

the definition of $\bar{M}_t^{F,frm}$ can be found in (30), it represents the asset's value of all the FRMs in the intermediary's balance sheet. Equation (160) shows that the intermediary picks the fraction s_t by balancing the (uncertain) net benefit of insuring an extra unit of the FRMs, $\mathcal{P}_{agg}^{frm,insured}(\mathcal{Z}_{t+1}) - \mathcal{P}_{agg}^{frm,uninsured}(\mathcal{Z}_{t+1})$, where $\mathcal{P}_{agg}^{frm,uninsured}(\mathcal{Z}_{t+1})$ represents the payoff that the intermediary would have gotten even without insuring the balance sheet. The intermediary needs to consider the marginal cost of insuring one extra unit of the balance sheet, $\phi^{fee}\cdot\bar{M}_t^{F,frm}$, which is a certain payment even at $t+1$. As with the other first order conditions, the bank accounts for the effect that these choices have on the regulatory constraint.

7.19 Mortgage Pricing Function Derivatives

In this section, I compute the mortgage pricing function derivatives introduced in equations (17) and (18). These functions are necessary to solve the first order conditions (111), (113), (114), (119), (120) from the borrower's problem. To compute the mortgage pricing function derivatives I need differentiate the financial intermediary's first order conditions related to the new originations (i.e when mortgages are priced) with respect to all the elements in $\alpha_t^{B,r}$.

In particular, to compute the the FRMs pricing derivatives (17) I need to differentiate (158) with respect to all the elements in $\alpha_t^{B,r}$. To compute the the ARMs pricing derivatives (18) I need to differentiate (159) with respect to all the elements in $\alpha_t^{B,r}$.

7.19.1 Mortgage Pricing Function Derivatives: FRMs

The first step is multiplying (158) by $m_t^{r,frm}$. I also slightly redefine the auxiliary variables used to compute (158).

$$\hat{\upsilon}_{t+1}^{m,r,frm,1} = \upsilon_{t+1}^{m,r,frm,1}(\mathcal{Z}_{t+1}) \cdot m_t^{r,frm}$$

$$\hat{\upsilon}_{t+1}^{m,r,frm,2} = \upsilon_{t+1}^{m,r,frm,2}(\mathcal{Z}_{t+1}) \cdot m_t^{r,frm}$$

$$\hat{\upsilon}_{t+1}^{m,r,frm,3} = \upsilon_{t+1}^{m,r,frm,3}(\mathcal{Z}_{t+1}) \cdot m_t^{r,frm}$$

$$\hat{\upsilon}_{t+1}^{m,r,frm,4} = F_\epsilon(\hat{\epsilon}_{t+1}^{r,frm}) \cdot m_t^{r,frm}$$

The FOC (158) can be rewritten as,

$$
\begin{aligned}
&E_t\Bigg[\mathcal{S}^F_{t,t+1}\Bigg\{(1-s_t)\Bigg(\hat{v}^{m,r,frm,1}_{t+1}\cdot\iota^{frm}\left(\alpha^{B,r}_t,\mathcal{Z}_t\right)+\hat{v}^{m,r,frm,2}_{t+1}+\hat{v}^{m,r,frm,3}_{t+1}\Bigg)\\
&+s_t\Bigg(\hat{v}^{m,r,frm,1}_{t+1}\cdot\iota^{frm}\left(\alpha^{B,r}_t,\mathcal{Z}_t\right)+\hat{v}^{m,r,frm,3}_{t+1}+\hat{v}^{m,r,frm,4}_{t+1}(\hat{\epsilon}^{r,frm}_{t+1})\cdot\left(1+\iota^{frm}\left(\alpha^{B,r}_t,\mathcal{Z}_t\right)\right)\Bigg)-\phi^{fee}\cdot s_t\cdot m^{r,frm}_t\Bigg\}\Bigg]\\
&+\kappa^I_t\Bigg[\left(1-\overline{e}^{frm,uninsured}\right)(1-s_t)\Bigg(\hat{v}^{m,r,frm,1}_{t+1}(\underline{z}_t)\cdot\iota^{frm}\left(\alpha^{B,r}_t,\underline{z}_t\right)+\hat{v}^{m,r,frm,2}_{t+1}(\underline{z}_t)+\hat{v}^{m,r,frm,3}_{t+1}(\underline{z}_t)\Bigg)\\
&+\left(1-\overline{e}^{frm,insured}\right)\cdot s_t\cdot\Bigg(\hat{v}^{m,r,frm,1}_{t+1}(\underline{z}_t)\cdot\iota^{frm}\left(\alpha^{B,r}_t,\underline{z}_t\right)+\hat{v}^{m,r,frm,3}_{t+1}(\underline{z}_t)+\hat{v}^{m,r,frm,4}_{t+1}(\underline{z}_t)\left(1+\iota^{frm}\left(\underline{z}_t\right)\right)\Bigg)\\
&-\phi^{fee}\cdot s_t\cdot m^{f,frm}_t\Bigg]=\left(1+\frac{\partial C^{orig}(\cdot)}{\partial M^{F,r,frm}_t}\right)\cdot m^{f,frm}_t
\end{aligned}
\tag{161}
$$

I can compute the derivatives of the mortgage pricing functions by differentiating (161) with respect to $a\in\alpha^{B,r}_t$, where I highlight the terms that include the mortgage pricing function derivative itself,

$\left(\frac{\partial \iota^{frm}(\alpha_t^{B,r},\mathcal{Z}_t)}{\partial a}\right)$.

$$
\begin{aligned}
&E_t\Bigg[\mathcal{S}^F_{t,t+1}\Bigg\{(1-s_t)\cdot\left(\frac{\partial \hat{v}^{m,r,frm,2}_{t+1}}{\partial \mu}+\frac{\partial \hat{v}^{m,r,frm,3}_{t+1}}{\partial \mu}\right)+s_t\cdot\left(\frac{\partial \hat{v}^{m,r,frm,3}_{t+1}}{\partial \mu}+\frac{\partial \hat{v}^{m,r,frm,4}_{t+1}}{\partial \mu}\right)\\
&+\Bigg((1-s_t)\cdot\left(1-F_\epsilon(\hat{\epsilon}^{r,frm}_{t+1})\right)+s_t\Bigg)\cdot(1+\delta\cdot p^{F,I,frm}_{t+1})\cdot\Bigg(\iota^{frm}(\alpha_t^{B,r},\mathcal{Z}_t)\cdot\left(\frac{\partial m_t^{r,frm}}{\partial \mu}\right)\\
&+m_t^{r,frm}\left(\frac{\partial \iota^{frm}(\alpha_t^{B,r},\mathcal{Z}_t)}{\partial \mu}\right)\Bigg)-(1-s_t)\cdot f_\epsilon(\hat{\epsilon}^{r,frm}_{t+1})\left(\frac{\partial \hat{\epsilon}^{r,frm}_{t+1}}{\partial \mu}\right)\cdot(1+\delta\cdot p^{F,I,frm}_{t+1})\cdot\iota^{frm}(\alpha_t^{B,r},\mathcal{Z}_t)m_t^{r,frm}\\
&-\phi^{fee}\cdot s_t\cdot\left(\frac{\partial m_t^{r,frm}}{\partial \mu}\right)\Bigg\}\Bigg]\\
&+\kappa_t^I\Bigg[\left(1-\overline{e}^{frm,uninsured}\right)\cdot(1-s_t)\cdot\Bigg\{\left(\frac{\partial \hat{v}^{m,r,frm,2}_{t+1}}{\partial \mu}+\frac{\partial \hat{v}^{m,r,frm,3}_{t+1}}{\partial \mu}\right)\\
&+\left(1-F_\epsilon(\hat{\epsilon}^{r,frm}_{t+1}(\underline{z}_t))\right)\cdot\left(1+\delta\cdot p^{F,I,frm}_{t+1}(\underline{z}_t)\right)\left(\iota^{frm}(\alpha_t^{B,r},\mathcal{Z}_t)\left(\frac{\partial m_t^{r,frm}}{\partial \mu}\right)+m_t^{r,frm}\left(\frac{\partial \iota^{frm}(\alpha_t^{B,r},\mathcal{Z}_t)}{\partial \mu}\right)\right)\\
&-f_\epsilon(\hat{\epsilon}^{r,frm}_{t+1}(\underline{z}_t))\left(\frac{\partial \hat{\epsilon}^{r,frm}_{t+1}(\underline{z}_t)}{\partial \mu}\right)\cdot\left(1+\delta\cdot p^{F,I,frm}_{t+1}(\underline{z}_t)\right)\cdot\iota^{frm}(\alpha_t^{B,r},\mathcal{Z}_t)m_t^{r,frm}\Bigg\}\\
&+(1-e^{frm,insured})\cdot s_t\cdot\Bigg\{\left(\frac{\partial \hat{v}^{m,r,frm,3}_{t+1}}{\partial \mu}+\frac{\partial \hat{v}^{m,r,frm,4}_{t+1}}{\partial \mu}\right)\\
&+\cdot\left(1+\delta\cdot p^{F,I,frm}_{t+1}(\underline{z}_t)\right)\cdot\left(\iota^{frm}(\alpha_t^{B,r},\mathcal{Z}_t)\left(\frac{\partial m_t^{r,frm}}{\partial \mu}\right)+m_t^{r,frm}\left(\frac{\partial \iota^{frm}(\alpha_t^{B,r},\mathcal{Z}_t)}{\partial \mu}\right)\right)\Bigg\}\\
&-\phi^{fee}\cdot s_t\cdot\left(\frac{\partial m_t^{r,frm}}{\partial \mu}\right)\Bigg]=\frac{\partial\left(1+\frac{\partial C^{orig}(\cdot)}{\partial M_t^{F,r,frm}}\right)}{\partial \mu}
\end{aligned}
\tag{162}
$$

Specific Derivatives - FRMs

The derivatives of the auxiliary variables with respect to $a \in \alpha_t^{B,r}$

$$
\begin{aligned}
\frac{\hat{v}^{m,r,frm,2}_{t+1}}{\partial a}&=F_\epsilon(\hat{\epsilon}^{r,frm}_{t+1})(1-\zeta)\cdot p^h_{t+1}\cdot(1-\delta^h)\epsilon^{minus,r,frm}_{t+1}\left[\frac{\partial h_t^B}{\partial a}\chi_t^{r,frm}+h_t^B\frac{\partial \chi_t^{r,frm}}{\partial a}\right]\\
&\quad+f_\epsilon(\hat{\epsilon}^{r,frm}_{t+1})\cdot\hat{\epsilon}^{r,frm}_{t+1}\cdot\left(\frac{\partial \hat{\epsilon}^{r,frm}_{t+1}}{\partial a}\right)(1-\zeta)\cdot p^h_{t+1}\cdot(1-\delta^h)h_t^B\chi_t^{r,frm}\\
\frac{\partial \hat{v}^{m,r,frm,3}_{t+1}}{\partial a}&=\left((1-\delta)+\delta\cdot p^{F,m,frm}_{t+1}\right)\left\{\left(1-F_\epsilon(\hat{\epsilon}^{r,frm}_{t+1})\right)\left(\frac{\partial m_t^{r,frm}}{\partial a}\right)-f_\epsilon(\hat{\epsilon}^{r,frm}_{t+1})\left(\frac{\partial \hat{\epsilon}^{r,frm}_{t+1}}{\partial a}\right)m_t^{r,frm}\right\}\\
\frac{\partial \hat{v}^{m,r,frm,4}_{t+1}}{\partial a}&=F_\epsilon(\hat{\epsilon}^{r,frm}_{t+1})\left(\frac{\partial m_t^{r,arm}}{\partial a}\right)+f_\epsilon(\hat{\epsilon}^{r,frm}_{t+1})\left(\frac{\partial \hat{\epsilon}^{r,frm}_{t+1}}{\partial a}\right)m_t^{r,frm}
\end{aligned}
$$

Using the default thresholds expression for the FRMs derived in (**??**),

$$\hat{\epsilon}^{r,frm}_{t+1} \cdot \underbrace{\left(1-\delta^h\right) p^{B,h}_{t+1} \cdot h^B_t \cdot \chi^{r,frm}_t}_{\text{Housing value funded with an FRM}}$$
$$= \underbrace{\left(\iota^{frm}(\alpha^{B,r}_t, \mathcal{Z}_t) \cdot (1+\delta \cdot p^{B,I,frm}_{t+1}) + (1-\delta) + \delta \cdot p^{B,m,frm}_{t+1}\right) m^{r,frm}_t}_{\text{Mortgage Market Value = Cost of not defaulting on the FRM}} - \underbrace{\lambda^{B,frm} w^{B,nh,r}_{t+1}}_{\text{Cost of defaulting on the FRM}},$$

I can find analytical solutions for the derivatives of the default thresholds with respect to $a \in \alpha^{B,r}_t$

$$\frac{\partial \hat{\epsilon}^{r,frm}_{t+1}}{\partial h^B_t} = -\frac{\hat{\epsilon}^{r,frm}_{t+1}}{h^B_t}$$
$$\frac{\partial \hat{\epsilon}^{r,frm}_{t+1}}{\partial n^B_t} = -\frac{\lambda^{B,frm} \cdot \frac{\partial w^{B,nh,r}_{t+1}}{\partial n^B_t}}{(1-\delta^h)\, p^{B,h}_{t+1} \cdot h^B_t \cdot \chi^{r,frm}_t}$$
$$\frac{\partial \hat{\epsilon}^{r,frm}_{t+1}}{\partial d^{B,r}_t} = -\frac{\lambda^{B,frm} \cdot \frac{\partial w^{B,nh,r}_{t+1}}{\partial d^{B,r}_t}}{(1-\delta^h)\, p^{B,h}_{t+1} \cdot h^B_t \cdot \chi^{r,frm}_t}$$
$$\frac{\partial \hat{\epsilon}^{r,frm}_{t+1}}{\partial m^{r,frm}_t} = \frac{\left(\iota^{frm}(\alpha^{B,r}_t, \mathcal{Z}_t) \cdot (1+\delta \cdot p^{B,I,frm}_{t+1}) + (1-\delta) + \delta \cdot p^{B,m,frm}_{t+1}\right)}{(1-\delta^h)\, p^{B,h}_{t+1} \cdot h^B_t \cdot \chi^{r,frm}_t}$$
$$\frac{\partial \hat{\epsilon}^{r,frm}_{t+1}}{\partial m^{r,arm}_t} = 0$$

This fully characterize the terms involving the FRMs pricing functions in the first order conditions (111), (113), (114), (119), (120). Next, I do the same for the ARMs.

7.19.2 Mortgage Pricing Function Derivatives: ARMs

The first step is multiplying (159) by $m^{r,arm}_t$. I also slightly redefine the auxiliary variables used to compute (159).

$$\hat{v}^{m,r,arm,1}_{t+1} = v^{m,r,arm,1}_{t+1}(\mathcal{Z}_{t+1}) \cdot m^{r,arm}_t$$

$$\hat{v}^{m,r,arm,2}_{t+1} = v^{m,r,arm,2}_{t+1}(\mathcal{Z}_{t+1}) \cdot m^{r,arm}_t$$

$$\hat{v}^{m,r,arm,3}_{t+1} = v^{m,r,arm,3}_{t+1}(\mathcal{Z}_{t+1}) \cdot m^{r,arm}_t$$

The FOC (159) can be rewritten as,

$$
\begin{aligned}
& E_t\left[\mathcal{S}^F_{t,t+1}\left\{\hat{v}^{m,r,arm,1}_{t+1}\cdot\left(i(\mathcal{Z}_{t+1})+spread^{arm}(\alpha^{B,r}_t,\mathcal{Z}_t)\right)+\hat{v}^{m,r,arm,2}_{t+1}+\hat{v}^{m,r,arm,3}_{t+1}\right\}\right] \\
& +\kappa^I_t\left[(1-\bar{e}^{arm})\left\{\hat{v}^{m,r,arm,1}_{t+1}(\underline{z}_t)\left(i(\underline{z}_t)+spread^{arm}(\alpha^{B,r}_t,\underline{z}_t)\right)+\hat{v}^{m,r,arm,2}_{t+1}(\underline{z}_t)+\hat{v}^{m,r,arm,3}_{t+1}(\underline{z}_t)\right]\right. \\
& =\left(1+\frac{\partial C^{orig}(\cdot)}{\partial M^{F,r,arm}_t}\right)\cdot m^{r,arm}_t
\end{aligned}
\tag{163}
$$

I can compute the derivatives of the mortgage pricing functions by differentiating (163) with respect to $a\in\alpha^{B,r}_t$, where I highlight the terms that include the mortgage pricing function derivative itself, $\left(\frac{\partial spread^{arm}(\alpha^{B,r}_t,\mathcal{Z}_t)}{\partial a}\right)$.

$$
\begin{aligned}
& E_t\left[\mathcal{S}^F_{t,t+1}\left\{\frac{\partial\hat{v}^{m,r,arm,2}_{t+1}}{\partial\mu}+\frac{\partial\hat{v}^{m,r,arm,3}_{t+1}}{\partial\mu}\right.\right. \\
& +\left(1-F_\epsilon(\hat{\epsilon}^{r,arm}_{t+1})\right)\cdot\left(1+\delta\cdot p^{F,I,arm}_{t+1}\right)\left[\left(i(\mathcal{Z}_{t+1})+spread^{arm}(\alpha^{B,r}_t,\mathcal{Z}_t)\right)\left(\frac{\partial m^{r,arm}_t}{\partial\mu}\right)\right. \\
& \left.+m^{r,arm}_t\left(\frac{\partial spread^{arm}(\alpha^{B,r}_t,\mathcal{Z}_t)}{\partial\mu}\right)\right] \\
& \left.\left.-f_\epsilon(\hat{\epsilon}^{r,arm}_{t+1})\left(\frac{\partial\hat{\epsilon}^{r,arm}_{t+1}}{\partial\mu}\right)\cdot\left(1+\delta\cdot p^{F,I,arm}_{t+1}\right)\cdot\left(i(\mathcal{Z}_{t+1})+spread^{arm}(\alpha^{B,r}_t,\mathcal{Z}_t)\right)\cdot m^{r,arm}_t\right\}\right] \\
& +\kappa^I_t(1-e^{arm})\left[\frac{\partial\hat{v}^{m,r,arm,2}_{t+1}(\underline{z}_t)}{\partial\mu}+\frac{\partial\hat{v}^{m,r,arm,3}_{t+1}(\underline{z}_t)}{\partial\mu}\right. \\
& +\left(1-F_\epsilon(\hat{\epsilon}^{r,arm}_{t+1}(\underline{z}_t))\right)\cdot\left(1+\delta\cdot p^{F,I,arm}_{t+1}(\underline{z}_t)\right)\left[\left(i(\underline{z}_t))+spread^{arm}(\alpha^{B,r}_t,\mathcal{Z}_t)\right)\left(\frac{\partial m^{r,arm}_t}{\partial\mu}\right)\right. \\
& \left.+m^{r,arm}_t\left(\frac{\partial spread^{arm}(\alpha^{B,r}_t,\mathcal{Z}_t)}{\partial\mu}\right)\right] \\
& \left.-f_\epsilon(\hat{\epsilon}^{r,arm}_{t+1}(\underline{z}_t))\left(\frac{\partial\hat{\epsilon}^{r,arm}_{t+1}(\underline{z}_t)}{\partial\mu}\right)\cdot\left(1+\delta\cdot p^{F,I,arm}_{t+1}(\underline{z}_t)\right)\left(i(\underline{z}_t))+spread^{arm}(\alpha^{B,r}_t,\mathcal{Z}_t)\right)m^{r,arm}_t\right]
\end{aligned}
\tag{164}
$$

Specific Derivatives - ARMs

The derivatives of the auxiliary variables with respect to $a\in\alpha^{B,r}_t$

$$
\begin{aligned}
\frac{\hat{v}_{t+1}^{m,r,arm,2}}{\partial a} &= F_\epsilon(\hat{\epsilon}_{t+1}^{r,arm})(1-\zeta)\cdot p_{t+1}^{h}\cdot(1-\delta^h)\epsilon_{t+1}^{minus,r,arm}\left[\frac{\partial h_t^B}{\partial a}\chi_t^{r,arm}+h_t^B\frac{\partial \chi_t^{r,arm}}{\partial a}\right] \\
&+ f_\epsilon(\hat{\epsilon}_{t+1}^{r,arm})\cdot\hat{\epsilon}_{t+1}^{r,arm}\cdot\left(\frac{\partial\hat{\epsilon}_{t+1}^{r,arm}}{\partial a}\right)(1-\zeta)\cdot p_{t+1}^{h}\cdot(1-\delta^h)h_t^B\chi_t^{r,arm} \\
\frac{\partial \hat{v}_{t+1}^{m,r,arm,3}}{\partial a} &= \left((1-\delta)+\delta\cdot p_{t+1}^{F,m,arm}\right)\left\{\left(1-F_\epsilon(\hat{\epsilon}_{t+1}^{r,arm})\right)\left(\frac{\partial m_t^{r,arm}}{\partial a}\right)-f_\epsilon(\hat{\epsilon}_{t+1}^{r,arm})\left(\frac{\partial\hat{\epsilon}_{t+1}^{r,arm}}{\partial a}\right)m_t^{r,arm}\right\}
\end{aligned}
$$

Using the default thresholds expression for the ARMs derived in (25),

$$
\begin{aligned}
&\hat{\epsilon}_{t+1}^{l,arm}\cdot\underbrace{\left(1-\delta^h\right)p_{t+1}^{B,h}\cdot h_t^B\cdot\chi_t^{l,arm}}_{\text{Housing value funded with an ARM}} \\
&= \underbrace{\left(\left(spread^{arm}\left(\alpha_t^{B,r},\mathcal{Z}_t\right)+i(\mathcal{Z}_{t+1})\right)\cdot(1+\delta\cdot p_{t+1}^{B,I,arm})+(1-\delta)+\delta\cdot p_{t+1}^{B,m,arm}\right)m_t^{l,arm}}_{\text{Mortgage Market Value = Cost of not defaulting on the ARM}} \\
&- \underbrace{\lambda^{B,arm}w_{t+1}^{B,nh,l}}_{\text{Cost of defaulting on the ARM}}
\end{aligned}
$$

I can find analytical solutions for the derivatives of the default thresholds with respect to $a\in\alpha_t^{B,r}$

$$
\begin{aligned}
\frac{\partial\hat{\epsilon}_{t+1}^{r,arm}}{\partial h_t^B} &= -\frac{\hat{\epsilon}_{t+1}^{r,arm}}{h_t^B} \\
\frac{\partial\hat{\epsilon}_{t+1}^{r,arm}}{\partial n_t^B} &= -\frac{\lambda^{B,arm}\cdot\frac{\partial w_{t+1}^{B,nh,r}}{\partial n_t^B}}{\left(1-\delta^h\right)p_{t+1}^{B,h}\cdot h_t^B\cdot\chi_t^{r,arm}} \\
\frac{\partial\hat{\epsilon}_{t+1}^{r,arm}}{\partial d_t^{B,r}} &= -\frac{\lambda^{B,arm}\cdot\frac{\partial w_{t+1}^{B,nh,r}}{\partial d_t^{B,r}}}{\left(1-\delta^h\right)p_{t+1}^{B,h}\cdot h_t^B\cdot\chi_t^{r,arm}} \\
\frac{\partial\hat{\epsilon}_{t+1}^{r,arm}}{\partial m_t^{r,arm}} &= \frac{\left(\left(spread^{arm}\left(\alpha_t^{B,r},\mathcal{Z}_t\right)+i(\mathcal{Z}_{t+1})\right)\cdot(1+\delta\cdot p_{t+1}^{B,I,arm})+(1-\delta)+\delta\cdot p_{t+1}^{B,m,arm}\right)}{\left(1-\delta^h\right)p_{t+1}^{B,h}\cdot h_t^B\cdot\chi_t^{r,arm}} \\
\frac{\partial\hat{\epsilon}_{t+1}^{r,arm}}{\partial m_t^{r,frm}} &= 0
\end{aligned}
$$

7.20 Proof: Homogeneity of degree zero in the Mortgage Pricing Functions

Proposition 2 The mortgage pricing function are homogeneous of degree zero in the borrower's portfolio choices $\alpha_t^{B,r}$. That is, consider some borrower 1 that optimally chooses $\alpha_t^{B1,r}$ at the refinancing stage such the pair of mortgages prices offered among the two entire menu contracts are,

$$\left\{\iota^{frm}(\alpha_t^{B1,r},\mathcal{Z}_t), spread^{arm}(\alpha_t^{B1,r},\mathcal{Z}_t)\right\}$$

then consider some borrower 2 that optimally chooses $\alpha_t^{B2,r} = k \cdot \alpha_t^{B1,r}$, for some $k > 0$, at the refinancing stage such the pair of mortgages prices offered among the two entire menu contracts are,

$$\left\{\iota^{frm}(\alpha_t^{B2,r},\mathcal{Z}_t), spread^{arm}(\alpha_t^{B2,r},\mathcal{Z}_t)\right\}$$

The mortgage pricing function are homogeneous of degree zero in the borrower's portfolio choices $\alpha_t^{B,r}$ if the following is true,

$$\begin{aligned}\iota^{frm}(\alpha_t^{B1,r},\mathcal{Z}_t) &= \iota^{frm}(\alpha_t^{B2,r},\mathcal{Z}_t)\\ spread^{arm}(\alpha_t^{B1,r},\mathcal{Z}_t) &= spread^{arm}(\alpha_t^{B2,r},\mathcal{Z}_t)\end{aligned}$$

Proof:

I will conjecture and then verify that degree zero in the borrower's portfolio choices $\alpha_t^{B,r}$,

$$\begin{aligned}\iota^{frm}(\alpha_t^{B,r},\mathcal{Z}_t) &= \iota^{frm}(k\cdot\alpha_t^{B,r},\mathcal{Z}_t)\\ spread^{arm}(\alpha_t^{B,r},\mathcal{Z}_t) &= spread^{arm}(k\cdot\alpha_t^{B,r},\mathcal{Z}_t)\end{aligned} \tag{165}$$

for any $k > 0$.

Step 1: Proof the default thresholds for the refinancers are homogeneous of degree zero

FRMs: the default thresholds for refinancers equals,

$$\begin{aligned}&\hat{\epsilon}_{t+1}^{r,frm}(\alpha_t^{B,r})\\ &= \frac{\left(\iota^{frm}(\alpha_t^{B,r},\mathcal{Z}_t)\cdot(1+\delta\cdot p_{t+1}^{B,I,frm}) + (1-\delta) + \delta\cdot p_{t+1}^{B,m,frm}\right) m_t^{r,frm} - \lambda^{B,frm} w_{t+1}^{B,nh,r}(\alpha_t^{B,r})}{(1-\delta^h)\, p_{t+1}^{B,h}\cdot h_t^B\cdot\chi_t^{r,frm}}\end{aligned}$$

derived in (103), and where I make its dependence on $\alpha_t^{B,r}$ explicit. The definition of non-housing wealth for refinancers (see (7)) is,

$$w_{t+1}^{B,nh,r}(\alpha_t^{B,r}) = (y_{t+1}^{B,n} + p_{t+1}^{B,n}) \cdot n_t^B + r_{t+1}^d \cdot d_t^{B,r}$$

Let $k > 0$, and define the default threshold for this alternative borrower, $\hat{\epsilon}_{t+1}^{r,frm}(k \cdot \alpha_t^{B,r})$ as,

$$\begin{aligned}
&\hat{\epsilon}_{t+1}^{r,frm}(k \cdot \alpha_t^{B,r}) \\
&= \frac{\left(\iota^{frm}(k \cdot \alpha_t^{B,r}, \mathcal{Z}_t) \cdot (1 + \delta \cdot p_{t+1}^{B,I,frm}) + (1-\delta) + \delta \cdot p_{t+1}^{B,m,frm}\right) k \cdot m_t^{r,frm} - \lambda^{B,frm} w_{t+1}^{B,nh,r}(k \cdot \alpha_t^{B,r})}{(1-\delta^h)\, p_{t+1}^{B,h} \cdot k \cdot h_t^B \cdot \chi_t^{r,frm}}
\end{aligned}$$

and non-housing wealth,

$$w_{t+1}^{B,nh,r}(k \cdot \alpha_t^{B,r}) = (y_{t+1}^{B,n} + p_{t+1}^{B,n}) \cdot k \cdot n_t^B + r_{t+1}^d \cdot k \cdot d_t^{B,r}$$

Using (165), and the fact that $w_{t+1}^{B,nh,r}(k \cdot \alpha_t^{B,r}) = k \cdot w_{t+1}^{B,nh,r}(\alpha_t^{B,r})$ we conclude that,

$$\hat{\epsilon}_{t+1}^{r,frm}(k \cdot \alpha_t^{B,r}) = \hat{\epsilon}_{t+1}^{r,frm}(\alpha_t^{B,r}) \tag{166}$$

ARMs: the default thresholds for refinancers equals,

$$\begin{aligned}
&\hat{\epsilon}_{t+1}^{r,arm}(\alpha_t^{B,r}) \\
&= \frac{\left(\left(spread^{arm}\left(\alpha_t^{B,r}, \mathcal{Z}_t\right) + i(\mathcal{Z}_{t+1})\right) \cdot (1 + \delta \cdot p_{t+1}^{B,I,arm}) + (1-\delta) + \delta \cdot p_{t+1}^{B,m,arm}\right) m_t^{l,arm}}{(1-\delta^h)\, p_{t+1}^{B,h} \cdot h_t^B \cdot \chi_t^{l,arm}} \\
&- \frac{\lambda^{B,arm} w_{t+1}^{B,nh,l}(\alpha_t^{B,r})}{(1-\delta^h)\, p_{t+1}^{B,h} \cdot h_t^B \cdot \chi_t^{l,arm}}
\end{aligned}$$

derived in (100), and where I make its dependence on $\alpha_t^{B,r}$ explicit. The definition of non-housing wealth is the same as with FRMs.

Let $k > 0$, and define the default threshold for this alternative borrower, $\hat{\epsilon}_{t+1}^{r,arm}(k \cdot \alpha_t^{B,r})$ as,

$$
\begin{aligned}
&\hat{\epsilon}_{t+1}^{r,arm}(k \cdot \alpha_t^{B,r}) \\
&= \frac{\left(\left(spread^{arm}\left(k \cdot \alpha_t^{B,r}, \mathcal{Z}_t\right) + i(\mathcal{Z}_{t+1})\right) \cdot (1 + \delta \cdot p_{t+1}^{B,I,arm}) + (1-\delta) + \delta \cdot p_{t+1}^{B,m,arm}\right) k \cdot m_t^{l,arm}}{(1-\delta^h)\, p_{t+1}^{B,h} \cdot k \cdot h_t^B \cdot \chi_t^{l,arm}} \\
&- \frac{\lambda^{B,arm} w_{t+1}^{B,nh,l}(k \cdot \alpha_t^{B,r})}{(1-\delta^h)\, p_{t+1}^{B,h} \cdot k \cdot h_t^B \cdot \chi_t^{l,arm}}
\end{aligned}
$$

Using (165), and the fact that $w_{t+1}^{B,nh,r}(k \cdot \alpha_t^{B,r}) = k \cdot w_{t+1}^{B,nh,r}(\alpha_t^{B,r})$ we conclude that,

$$
\hat{\epsilon}_{t+1}^{r,arm}(k \cdot \alpha_t^{B,r}) = \hat{\epsilon}_{t+1}^{r,arm}(\alpha_t^{B,r}) \tag{167}
$$

Step 2: Proof the mortgage pricing functions are homogeneous of degree zero

FRMs: To show this, I will use the FOC of the financial intermediary with respect to new FRM originations, (158). Notice the following is true about this equation,

1. Using the results from equation (167)[116],

$$
\hat{\epsilon}_{t+1}^{r,frm}(k \cdot \alpha_t^{B,r}) = \hat{\epsilon}_{t+1}^{r,frm}(\alpha_t^{B,r}) \implies F\left(\hat{\epsilon}_{t+1}^{r,frm}(k \cdot \alpha_t^{B,r})\right) = F\left(\hat{\epsilon}_{t+1}^{r,frm}(\alpha_t^{B,r})\right)
$$

2. The auxiliary variables $\upsilon_{t+1}^{m,r,frm,1}(\mathcal{Z}_{t+1})$ and $\upsilon_{t+1}^{m,r,frm,3}(\mathcal{Z}_{t+1})$ depend on borrower's individual choices uniquely from the default threshold, hence,

$$
\begin{aligned}
\upsilon_{t+1}^{m,r,frm,1}(k \cdot \alpha_t^{B,r}, \mathcal{Z}_{t+1}) &= \upsilon_{t+1}^{m,r,frm,1}(\alpha_t^{B,r}, \mathcal{Z}_{t+1}) \\
\upsilon_{t+1}^{m,r,frm,3}(k \cdot \alpha_t^{B,r}, \mathcal{Z}_{t+1}) &= \upsilon_{t+1}^{m,r,frm,3}(\alpha_t^{B,r}, \mathcal{Z}_{t+1})
\end{aligned}
$$

3. The auxiliary variable $\upsilon_{t+1}^{m,r,frm,2}(\mathcal{Z}_{t+1})$ does depend on the ratio $\frac{h_t^B}{m_t^{r,frm}}$ (and the default threshold) which implies that,

$$
\frac{k \cdot h_t^B}{k \cdot m_t^{r,frm}} \frac{h_t^B}{m_t^{r,frm}} \implies \upsilon_{t+1}^{m,r,frm,2}(k \cdot \alpha_t^{B,r}, \mathcal{Z}_{t+1}) = \upsilon_{t+1}^{m,r,frm,2}(\alpha_t^{B,r}, \mathcal{Z}_{t+1})
$$

[116] In reality, these payoff functions represented by the auxiliary variables are integrals and what we are showing here is that the integration limits do not change for any $k > 0$.

4. I need to assume that the function $C^{orig}(\alpha_t^{B,r}, \mathcal{Z}_{t+1})$ is homogeneous of degree zero,

$$C^{orig}(k \cdot \alpha_t^{B,r}, \mathcal{Z}_{t+1}) = C^{orig}(\alpha_t^{B,r}, \mathcal{Z}_{t+1})$$

which I do, since it actually depends on the loan to value at origination, $\frac{m_t^{r,frm}}{h_t^B}$.

As a result from these 4 observations and analyzing the FOC of the financial intermediary with respect to new FRM originations, (158), if this equation holds exactly for some $\alpha_t^{B,r}$, then for it to hold for $k \cdot \alpha_t^{B,r}$, it is necessary that,

$$\iota^{frm}(\alpha_t^{B,r}, \mathcal{Z}_t) = \iota^{frm}(k \cdot \alpha_t^{B,r}, \mathcal{Z}_t)$$

which verifies the guess □.

ARMs: To show this, I will use the FOC of the financial intermediary with respect to new ARM originations, (159). The proof is analogous to the one for FRMs. Notice that the fact that the policy interest rate $i(\mathcal{Z}_{t+1})$ features in this FOC does not change anything. The I can conclude that if (159) holds exactly for some $\alpha_t^{B,r}$, then for it to hold for $k \cdot \alpha_t^{B,r}$, it is necessary that,

$$spread^{arm}(\alpha_t^{B,r}, \mathcal{Z}_t) = spread^{arm}(k \cdot \alpha_t^{B,r}, \mathcal{Z}_t)$$

which verifies the guess □.

7.21 Government payment of mortgage losses

The government is required to make a payment in period $t+1$ to cover the insured losses from the financial intermediary. Notice that this is an expenditure that depends on the state z_{t+1}. I define the required government payment, conditional on the mortgage status $l \in \{r, nr\}$ as,

$$\begin{aligned}\text{Gov}_{t+1}^{exp,l} = s_t \Bigg[q^l \Bigg(& \underbrace{F_\epsilon(\hat{\epsilon}_{t+1}^{l,frm}) \cdot \left(\frac{\text{payment}_{t+1}^{l,frm}}{m_t^{l,frm}} \right)}_{\text{FRMs mortgage payments}} + \underbrace{q^l \cdot F_\epsilon(\hat{\epsilon}_{t+1}^{l,frm})}_{\text{FRMs prepayments}} \\ & - \underbrace{F_\epsilon(\hat{\epsilon}_{t+1}^{l,frm}) \cdot (1-\zeta) \cdot p_{t+1}^h \left(1 - \delta^h\right) \left(\frac{\epsilon_{t+1}^{minus,l,frm} \cdot h_t^B}{m_t^{nr,frm}} \right) \chi_t^{l,frm} \Bigg) \cdot M_t^{F,l,frm}}_{\text{FRMs foreclosed housing}} \Bigg] \end{aligned} \tag{168}$$

The government covers the mortgage payments due at the moment of default and prepays the principal owed to the investor holder. The payment is net of the foreclosed housing value which I assume is transferred to the government once the mortgage is insured against credit risk. The total payment due is described in equation (169) which includes both new originations and old mortgage balances,

$$\mathrm{Gov}_{t+1}^{exp} = \sum_{l \in \{r,nr\}} \mathrm{Gov}_{t+1}^{exp,l} \tag{169}$$

Therefore, the mortgage guarantee pays the financial intermediary the difference between what a non-defaulter would owe at period $t+1$ and what the intermediary would have gotten if the mortgage was not insured and was defaulted in period $t+1$.

7.22 Rebate Equations

In this section I will describe in detail the second term Y_t^{rebate} of the total payoff Y_t^{mod} as described in (51) in the main text. This extra aggregate endowment income allows me to maintain the simplicity of an endowment economy, while satisfying the resource constraint, see equation (64). The first term incorporated to Y_t^{rebate} represents the non-housing wealth losses from those households that defaulted for which a fraction λ is vanished, it considers both refinancers and non-refinancers,

$$Y_t^{rebate,B} = \sum_{l \in \{r,nr\}} q_t^l \left(\lambda^{frm} F_\epsilon(\hat{\epsilon}_{t+1}^{l,frm}) + \lambda^{arm} F_\epsilon(\hat{\epsilon}_{t+1}^{l,arm}) \right) w_{t+1}^{B,nh,l} \tag{170}$$

the definition of $w_{t+1}^{B,nh,l}$ can be found in (7). Notice that the taxation term is not considered in these calculations, the assumption is that the same amount of taxes is paid by all borrowers equally no matter their default status[117]. The second term incorporated to Y_t^{rebate} represents the total resources lost when a fraction the house value owned by mortgagors are lost when they default, these are losses borne by the financial intermediary, again it considers both refinancers and non-refinancers,

[117] Adding the taxes in the rebate term would imply that defaulters pay less taxes.

$$
\begin{aligned}
Y_t^{rebate,I} &= \sum_{l\in\{r,nr\}} q_t^l \left[\zeta \cdot F_\epsilon(\hat{\epsilon}_{t+1}^{l,arm}) \cdot h_t^B \cdot p_{t+1}^h \cdot (1-\delta^h) \cdot \epsilon_{t+1}^{minus,l,arm} \chi_t^{l,arm}\right] \\
&+ \sum_{l\in\{r,nr\}} q_t^l \left[\zeta \cdot F_\epsilon(\hat{\epsilon}_{t+1}^{l,frm}) \cdot h_t^B \cdot p_{t+1}^h \cdot (1-\delta^h) \cdot \epsilon_{t+1}^{minus,l,frm} \chi_t^{l,frm}\right]
\end{aligned}
\tag{171}
$$

The total rebate is simply,

$$
Y_t^{rebate} = Y_t^{rebate,B} + Y_t^{rebate,I} \tag{172}
$$

7.23 Equilibrium Definition

Given a sequence of aggregate income and housing risk shocks $\{Y_t, \sigma(\epsilon_t)\}$, a competitive equilibrium is an allocation $\left\{H_t^B, N_t^B, D_t^{B,nr}, D_t^{B,r}, M_t^{nr,frm}, M_t^{nrarm}, M_t^{r,frm}, M_t^{rarm}, M_t^{I,frm}, M_t^{S,arm}\right\}$ for borrowers, $\left\{H_t^S, N_t^S, D_t^S, B_t^\xi\right\}$ for savers, $\left\{M_t^{F,r,frm}, M_t^{F,r,arm}, M_t^{F,nr,frm}, M_t^{F,nr,arm}, M_t^{F,I,frm}, M_t^{F,S,arm}, s_t, D_t, I_t\right\}$ for the financial intermediaries, the price vector for non-mortgage related variables $\{p_t^h, p_t^{B,n}, p_t^{S,n}, r_t^d, p_t^{S,\xi}, \tau_t\}$, and the price vector for mortgage related variables $\{p_t^{B,m,j}, p_t^{B,I,j}, p_t^{F,m,j}, p_t^{F,I,frm}\}_{j\in\{frm,arm\}}$ such that given these prices the borrowers solves problem (24), the saver solves problem (27), the financial intermediary solves (48), the government budget constraint (49) is satisfied, and the market clearing conditions (56)-(64) are satisfied. Additionally, the equilibrium requires to track the evolution of the model's state variables, I present expressions for the transition functions for each state variables in this Appendix 7.26.

7.24 Model Calibration

Table 1: Parameter values.

Parameter	Value	Interpretation	Basis
Preferences			
γ	1	Risk aversion	Standard in the literature
θ^S	0.14	Saver's housing weight in utility	Housing wealth to GDP ratio
θ^B	0.14	Borrower's housing weight in utility	Equal to θ^S.
β^S	0.96	Saver's discount factor	One-year real risk-free interest rate
β^B	0.93	Borrower's discount factor	Household debt to GDP ratio
ψ	0.006	Liquidity Preference	Liquidity Premium
Financial Intermediary			
$\bar{e}^{frm,insured}$	0.016	FRMs securitized through agency-MBS	Basel Regulation
$\bar{e}^{frm,uninsured}$	0.08	FRMs in balance sheet	Basel Regulation
$\bar{e}^{arm}$	0.08	ARMs in balance sheet	Basel Regulation
μ^{orig}	0.2	Origination Cost	30-year FRM spread over 10-year T-bill
τ	0.068	Target payout ratio	Bank dividend ratio (Elenev et al., 2021)
χ^F	1000	Equity issuance cost	Bank net payout rate (Elenev et al., 2021)
Housing			
$\bar{H}$	1	Housing Supply	Normalization
δ^h	0.023	Fixed housing maintenance costs	Depreciation of residential fixed assets
Transition $\sigma_\epsilon^{low} \to \sigma_\epsilon^{low}$	0.80	Persistence of low risk housing shock	Average duration of housing recession
Transition $\sigma_\epsilon^{high} \to \sigma_\epsilon^{high}$	0.95	Persistence of high risk housing shock	Unconditional prob. of housing recession
σ_ϵ^{low}	0.198	Housing shock value, normal times	Mortgage loss rate, normal times
σ_ϵ^{high}	0.252	Housing shock value, crisis times	Mortgage loss rate, crisis times
ζ	0.25	Foreclosure loss to bank	In line with Campbell et al. (2011)
Mortgages			
ϕ^{refi}	0.02	Borrower refinancing cost	Loan-to-value ratio for new originations
λ	0.01	Default Penalty	Loan-to-value ratio (all borrowers)
δ_{FRM}	0.97	Mortgage duration	Principal amortization on 30-yr FRM
q^r	0.18	Refinancing Probability	Refinancing share for 30-yr FRM
Endowment			
σ^y	0.023	Income shock standard deviation	Standard deviation income per capita
ρ^y	0.45	Income shock autocorrelation	Autocorrelation income per capita
ν	0.40	Borrower's income share	Income shares SCF, (Elenev et al., 2016)
Exogenous Policy Rate			
i^{low}	0.01%,	Exogenous policy rate during recession	Av. short-term policy (real) rates (recessions).
i^{mid}	1.1%,	Exogenous policy rate during normal times	Av. short-term policy (real) rates.
i^{high}	1.6%,	Exogenous policy rate during expansions	Av. short-term policy (real) rates (expansions).
ρ^I	0.45	Exogenous policy rate shock autocorrelation	Same to ρ^y
Government			
ϕ^{fee}	0.0063	Government insurance premium (g-fee)	Fraction of insured FRMs
ϕ^{fee}	0.15	Exogenous government expenditure	Government expenditure to GDP ratio

7.25 Model Results: Stationary Distribution

Table 2: Main Results - Endogenous Mortgage Choice (Part 1).

Experiment Name	**Baseline**		**Optimal ARM**		**Constrained ARM**	
Guarantee fee	$\phi^{fee}_{low} = 63$ bps		$\phi^{fee}_{high} = 500$ bps		$\phi^{fee}_{high} = 500$ bps	
Insured fraction of FRMs	70%		0%		0%	
	mean	std	mean	std	mean	std
ARM share (Originations)*						
ARM share (unconditional)	28.5%	0.8%	56.7%	0.3%	28.5%	0%
ARM share (crisis)$	26.7%	0.6%	56.4%	0.2%	28.5%	0%
ARM share (Outstanding)+						
ARM share (unconditional)	29.3%	0.4%	56.3%	0.1%	28.7%	0.05%
ARM share (crisis)	29.3%	0.5%	56.3%	0.1%	28.9%	0.04%
Borrower						
Borrower LTV (originations)	86.8%	2.0%	81.7%	2.5%	80.3%	2.6%
Borrower LTV (outstanding)	60.6%	1.3%	58.2%	1.4%	57.0%	1.5%
Borrower LTV (total)#	67.1%	1.2%	63.9%	1.3%	62.6%	1.4%
Borrower PTI (originations)	36.3%	0.6%	33.2%	0.7%	32.6%	0.8%
Borrower PTI (outstanding)	20.7%	0.4%	19.6%	0.3%	19.0%	0.4%
Borrower PTI (total)	23.4%	0.4%	21.9%	0.4%	21.2%	0.4%
Mortgage Default						
Unconditional						
Default Rate (originations)	4.3%	1.4%	1.0%	0.5%	1.1%	0.6%
Default Rate (outstanding)	1.3%	0.5%	0.7%	0.3%	0.7%	0.3%
Default Rate (total)	1.8%	0.6%	0.8%	0.3%	0.9%	0.4%
Crisis						
Default Rate (originations)	8.5%	2.1%	2.4%	0.9%	2.6%	1.0%
Default Rate (outstanding)	2.9%	0.4%	1.8%	0.3%	2.0%	0.3%
Default Rate (total)	3.9%	0.6%	1.9%	0.4%	2.1%	0.5%
Prices						
FRM Rate	3.6%	0.5%	4.4%	0.7%	4.6%	0.8%
ARM Rate	3.7%	0.3%	4.0%	0.3%	3.6%	0.2%
Spread (FRM-ARM)	-0.1%	0.2%	0.4%	0.2%	0.9%	0.6%
Risk-free Rate	1.8%	1.3%	1.7%	1.3%	1.7%	1.4%
House Price	1.971	0.045	1.950	0.045	1.942	0.047

The table reports unconditional means and standard deviations of the main outcome variables from a 20,000 period simulation for each of the three economies. The model in the first 2 columns has a mortgage guarantee fee of 63 bps which implies 70% of FRMs are insured. The model in columns 3 and 4 has has a mortgage guarantee fee of 500 bps which implies 0% of FRMs are insured, while allowing for the ARM share to be optimally chosen. The model in columns 5 and 6 is similar than the previous one but I do not allow for the ARM share to be optimally chosen, forcing it to match that of the model in columns 1 and 2.

* *Originations* refer to only new originations, in the model these are mortgages originated by refinancers.

\+ *Outstanding* refers to only old mortgage debt (excludes originations), in the model these are mortgages originated by non-refinancers.

\# *Total* incorporates all mortgage debt, new originations and old mortgage debt.

$ Crisis: Conditional on periods of low endowment realizations (Y_t) and high house price shock dispersion (σ_ϵ^{high}).

Table 3: Main Results - Endogenous Mortgage Choice (Part 2).

Experiment Name	**Baseline**		**Optimal ARM**		**Constrained ARM**	
Guarantee fee	$\phi^{fee}_{low} = 63$ bps		$\phi^{fee}_{high} = 500$ bps		$\phi^{fee}_{high} = 500$ bps	
Insured fraction of FRMs	70%		0%		0%	
	mean	std	mean	std	mean	std
Financial Intermediary						
Intermediary's Equity (unconditional)	0.053	0.013	0.059	0.012	0.058	0.012
Intermediary's Equity (crisis)$^{\$}$	0.032	0.009	0.037	0.006	0.036	0.006
Intermediary's Leverage	0.872	0.030	0.848	0.031	0.839	0.030
Multiplier CC	0.020	0.07	0.067	0.10	0.071	0.12
Max Value Multiplier CC	0.56	-	0.52	-	0.53	-
Secondary Market Prices						
PO ARM	0.897	0.020	0.895	0.020	0.897	0.021
PO FRM	0.906	0.017	0.898	0.020	0.887	0.021
PO ARM (crisis)	0.865	0.008	0.862	0.006	0.864	0.0006
PO FRM (crisis)	0.881	0.004	0.863	0.006	0.855	0.0007
Allocations						
**Relative to Baseline*						
Face value mortgage debt (originations)*	1	1	0.92	1.11	0.91	1.09
Face value mortgage debt (outstanding)$^{+}$	1	1	0.95	1.10	0.94	1.08
Saver's deposits	1	1	0.91	1.02	0.91	1.03
Consumption and Welfare						
**Relative (%) to Baseline*						
Borrower's Consumption	0%	0%	0.25%	-4.26%	0.21%	-3.90%
Saver's Consumption	0%	0%	-0.07%	-5.14%	-0.09%	-4.90%
Borrower's Value Function	0%	0%	0.14%	-4.50%	0.08%	-4.18%
Saver's Value Function	0%	0%	-0.02%	-5.06%	-0.03%	-4.76%

The table reports unconditional means and standard deviations of the main outcome variables from a 20,000 period simulation for each of the three economies. The model in the first 2 columns has a mortgage guarantee fee of 63 bps which implies 70% of FRMs are insured. The model in columns 3 and 4 has has a mortgage guarantee fee of 500 bps which implies 0% of FRMs are insured, while allowing for the ARM share to be optimally chosen. The model in columns 5 and 6 is similar than the previous one but I do not allow for the ARM share to be optimally chosen, forcing it to match that of the model in columns 1 and 2.

* *Originations* refer to only new originations, in the model these are mortgages originated by refinancers.

$^{+}$ *Outstanding* refers to only old mortgage debt (excludes originations), in the model these are mortgages originated by non-refinancers.

$^{\#}$ *Total* incorporates all mortgage debt, new originations and old mortgage debt.

$^{\$}$ Crisis: conditional on periods of low endowment realizations (Y_t) and high house price shock dispersion (σ^{high}_{ϵ}).

Table 4: Main Results - Default Rates by Mortgage Type.

Experiment Name	**Baseline**	**Optimal ARM**	**Constrained ARM**
Guarantee fee	$\phi^{fee}_{low} = 63$ bps	$\phi^{fee}_{high} = 500$ bps	$\phi^{fee}_{high} = 500$ bps
Insured fraction of FRMs	70%	0%	0%
	mean	mean	mean
ARMs			
Unconditional			
Default Rate (Originations)*	1.5%	1.0%	0.6%
Default Rate (Outstanding)+	0.5%	0.7%	0.4%
Default Rate (Total)#	0.7%	0.8%	0.4%
Crisis$			
Default Rate (Originations)	3.3%	2.4%	1.5%
Default Rate (Outstanding)	1.4%	1.8%	1.1%
Default Rate (Total)	1.8%	1.9%	1.2%
FRMs			
Unconditional			
Default Rate (Originations)	4.3%	0.7%	1.1%
Default Rate (Outstanding)	1.3%	0.4%	0.7%
Default Rate (Total)	1.8%	0.5%	0.8%
Crisis			
Default Rate (Originations)	8.6%	1.8%	2.6%
Default Rate (Outstanding)	2.9%	1.3%	2.0%
Default Rate (Total)	3.9%	1.4%	2.1%

The table reports unconditional means of the main outcome variables from a 20,000 period simulation for each of the three economies. The model in the first 2 columns has a mortgage guarantee fee of 63 bps which implies 70% of FRMs are insured. The model in columns 3 and 4 has has a mortgage guarantee fee of 500 bps which implies 0% of FRMs are insured, while allowing for the ARM share to be optimally chosen. The model in columns 5 and 6 is similar than the previous one but I do not allow for the ARM share to be optimally chosen, forcing it to match that of the model in columns 1 and 2.

* *Originations* refer to only new originations, in the model these are mortgages originated by refinancers.

\+ *Outstanding* refers to only old mortgage debt (excludes originations), in the model these are mortgages originated by non-refinancers.

\# *Total* incorporates all mortgage debt, new originations and old mortgage debt.

$ Crisis: conditional on periods of low endowment realizations (Y_t) and high house price shock dispersion (σ_ϵ^{high}).

7.26 State Variables and Transition Functions

There are seven state variables in the model. The post-default wealth for borrowers and savers, the post-default equity of the financial intermediary, the aggregate mortgage balances for both mortgage types, the aggregate mortgage interest payment for the FRMs, and the aggregate mortgage spread payment for the ARMs. In reality, the saver's wealth can be computed residually once I know the borrower's wealth and the bank's equity. In this section I provide expressions for the evolution of all these state variables, except for the bank's equity which was already computed in equation (43).

Borrower's Wealth Let $\epsilon_{t+1}^{minus,l,j} = E[\epsilon|\epsilon < \hat{\epsilon}_{t+1}^{l,j}]$ and $\epsilon_{t+1}^{plus,l,j} = E[\epsilon|\epsilon > \hat{\epsilon}_{t+1}^{l,j}]$ for $l \in \{nr, r\}$ and $j \in \{frm, arm\}$. Also define, $\epsilon_{t+1}^{mid,l,j} = E[\epsilon|\hat{\epsilon}_{t+1}^{l,k} > \epsilon > \hat{\epsilon}_{t+1}^{l,j}]$ where $k \in \{frm, arm\}$ and $k \neq j$.

The aggregate mortgage market value (as defined in (26)) can be written as,

$$MMV_{t+1}^{l,j} = \left(\frac{\text{payment}_{t+1}^{l,j}}{M_t^{l,j}} \cdot (1 + \delta \cdot p_{t+1}^{B,I,j}) + (1 - \delta^j) + \delta^j \cdot p_{t+1}^{B,m,j} \right) M_t^{l,j}.$$

where I abuse notation and in this section $\text{payment}_{t+1}^{l,j}$ will refer to the **aggregate** mortgage payments as described in the following equations,

$$\text{payment}_{t+1}^{l,frm} = \begin{cases} M_t^{I,frm} & \text{if } l = nr \\ \underbrace{\iota^{frm}\left(\alpha_t^{B,r}, \mathcal{Z}_t\right)}_{\text{Pricing function set at } t} M_t^{r,frm} & \text{if } l = r \end{cases}$$

$$\text{payment}_{t+1}^{l,arm} = \begin{cases} M_t^{S,arm} + \theta^m \cdot i_{t+1} \cdot M_t^{nr,arm} & \text{if } l = nr \\ \Big(\underbrace{spread^{arm}\left(\alpha_t^{B,r}, \mathcal{Z}_t\right)}_{\text{Pricing function set at } t} + \theta^m \cdot i_{t+1} \Big) \cdot M_t^{r,arm} & \text{if } l = r \end{cases}$$

where abusing notation again, the portfolio $\alpha_t^{B,r}$ in this section is composed of the **aggregate** asset choices at the refinancing stage. Define the aggregate wealth level $W_{t+1}^{B,l}$, conditional on the refinancing status l as,

$$
\begin{aligned}
W_{t+1}^{B,l} = \underbrace{q_t^l}_{\text{Refi Prob.}} \Bigg\{ & \underbrace{\mathbb{1}_{\{\hat{\epsilon}_{t+1}^{l,arm} < \hat{\epsilon}_{t+1}^{l,frm}\}}}_{\textbf{Only FRM default states}} \Bigg[\underbrace{\Big(1 - F_\epsilon(\hat{\epsilon}_{t+1}^{l,frm})\Big)}_{\textbf{No Default}} \cdot \Bigg[(y_{t+1}^B + p_{t+1}^{B,n}) \cdot 1 + D_t^{B,l} \cdot r_{t+1}^d \\
& - T_{t+1}^B + \Big(1 - \delta^h\Big) p_{t+1}^h \cdot \epsilon_{t+1}^{plus,l,frm} \cdot H_t^B - \sum_{j=\{arm,frm\}} MMV_{t+1}^{l,j} \Bigg] \\
& + \underbrace{\Big(F_\epsilon(\hat{\epsilon}_{t+1}^{l,frm}) - F_\epsilon(\hat{\epsilon}_{t+1}^{l,arm})\Big)}_{\textbf{FRM Default}} \Bigg[(1 - \lambda^{frm}) \Big((y_{t+1}^B + p_{t+1}^{B,n}) \cdot 1 + D_t^{B,l} \cdot r_{t+1}^d \Big) - T_{t+1}^B \\
& + \Big(1 - \delta^h\Big) p_{t+1}^h \cdot \epsilon_{t+1}^{mid,l,arm} \cdot H_t^B \cdot \chi_t^{l,arm} - MMV_{t+1}^{l,arm} \Bigg] \\
& + \underbrace{F_\epsilon(\hat{\epsilon}_{t+1}^{l,arm})}_{\textbf{Total Default}} \Big[(1 - \lambda^{frm} - \lambda^{arm}) \Big((y_{t+1}^B + p_{t+1}^{B,n}) \cdot 1 + D_t^{B,l} \cdot r_{t+1}^d \Big) - T_{t+1}^B \Big] \Bigg] \\
& + \underbrace{\mathbb{1}_{\{\hat{\epsilon}_{t+1}^{l,frm} < \hat{\epsilon}_{t+1}^{l,arm}\}}}_{\textbf{Only ARM default states}} \Bigg[\underbrace{\Big(1 - F_\epsilon(\hat{\epsilon}_{t+1}^{l,arm})\Big)}_{\textbf{No Default}} \cdot \Bigg[(y_{t+1}^B + p_{t+1}^{B,n}) \cdot 1 + D_t^{B,l} \cdot r_{t+1}^d \\
& - T_{t+1}^B + \Big(1 - \delta^h\Big) p_{t+1}^h \epsilon_{t+1}^{plus,l,arm} \cdot H_t^B - \sum_{j=\{arm,frm\}} MMV_{t+1}^{l,j} \Bigg] \\
& + \underbrace{\Big(F_\epsilon(\hat{\epsilon}_{t+1}^{l,arm}) - F_\epsilon(\hat{\epsilon}_{t+1}^{l,frm})\Big)}_{\textbf{ARM Default}} \Bigg[(1 - \lambda^{arm}) \Big((y_{t+1}^B + p_{t+1}^{B,n}) \cdot 1 + D_t^{B,l} \cdot r_{t+1}^d \Big) - T_{t+1}^B \\
& + \Big(1 - \delta^h\Big) p_{t+1}^h \cdot \epsilon_{t+1}^{mid,l,frm} \cdot H_t^B \cdot \chi_t^{l,frm} - MMV_{t+1}^{l,frm} \Bigg] \\
& + \underbrace{F_\epsilon(\hat{\epsilon}_{t+1}^{l,frm})}_{\textbf{Total Default}} \Big[(1 - \lambda^{frm} - \lambda^{arm}) \Big((y_{t+1}^B + p_{t+1}^{B,n}) \cdot 1 + D_t^{B,l} \cdot r_{t+1}^d \Big) - T_{t+1}^B \Big] \Bigg] \Bigg\}
\end{aligned}
\tag{173}
$$

The transition equation for the borrower's aggregate wealth is defined as,

$$
W_{t+1}^B = \sum_{l \in \{r,nr\}} W_{t+1}^{B,l} \tag{174}
$$

Mortgage Balances The evolution for the aggregate mortgage balances is described in the following equations,

$$M_{t+1}^{nr,frm} = \delta \cdot \left\{ \underbrace{q^{nr} \cdot \Big(1 - F(\hat{\epsilon}_t^{nr,frm})\Big) \cdot M_t^{nr,frm}}_{\textbf{FRM not refinanced}} + \underbrace{q^{r} \cdot \Big(1 - F(\hat{\epsilon}_t^{r,frm})\Big) \cdot M_t^{r,frm}}_{\textbf{FRM refinanced}} \right\} \tag{175}$$

$$M_{t+1}^{nr,arm} = \delta \cdot \left\{ \underbrace{q^{nr} \cdot (1 - F(\hat{\epsilon}_t^{nr,arm})) \cdot M_t^{nr,arm}}_{\textbf{ARM not refinanced}} + \underbrace{q^{r} \cdot (1 - F(\hat{\epsilon}_t^{r,arm})) \cdot M_t^{r,arm}}_{\textbf{ARM refinanced}} \right\} \tag{176}$$

Mortgage Payments The evolution for the aggregate mortgage interest payment on FRMs,

$$M_{t+1}^{I,frm} = \delta \cdot \left\{ \underbrace{q^{nr} \cdot \Big(1 - F(\hat{\epsilon}_t^{nr,frm})\Big) \cdot M_t^{I,frm}}_{\textbf{FRM not refinanced}} + \underbrace{q^{r} \cdot \Big(1 - F(\hat{\epsilon}_t^{r,frm})\Big) \cdot \iota^{frm} \left(\alpha_t^{B,r}, \mathcal{Z}_t\right) \cdot M_t^{r,frm}}_{\textbf{FRM refinanced}} \right\} \tag{177}$$

The evolution for the aggregate mortgage spread payment on ARMs,

$$M_{t+1}^{S,arm} = \delta \cdot \left\{ \underbrace{q^{nr} \cdot (1 - F(\hat{\epsilon}_t^{nr,arm})) \cdot M_t^{S,arm}}_{\textbf{ARM not refinanced}} + \underbrace{q^{r} \cdot (1 - F(\hat{\epsilon}_t^{r,arm})) \cdot spread^{arm} \left(\alpha_t^{B,r}, \mathcal{Z}_t\right) \cdot M_t^{r,arm}}_{\textbf{ARM refinanced}} \right\} \tag{178}$$